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Certification Duration For Food Assistance Programs: An Economic Model With An Application to WIC

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The Certification Duration (CD) model of this paper provides a formal economic framework for analyzing tradeoffs policymakers and program managers face when determining a client's certification duration. Due to income volatility, clients of food assistance programs who are income-eligible today are unlikely to be income-eligible forever: sooner or later, income can surpass the program's eligibility threshold. If relatively shorter CDs are chosen, benefits for those clients who have lost income-eligibility are terminated relatively sooner. That outcome results in budgetary savings and targets program benefits to eligible clients, promoting program integrity. However, a shorter CD raises recertification costs for the program agency and for clients, impinging on client access. Thus policymakers face tradeoffs among program goals. The paper models the policymaking tradeoff between two of these goals—program integrity and recertification cost—as a problem of minimization of expected discounted program costs across multiple recertification cycles. Changes in optimal CDs are related to changes in key parameters of the model, including the rate at which clients transition out of income-eligibility, the dollar value of program benefits, and recertification costs. Sets of numerical values for the model's parameters are used to simulate optimal CDs, measured in calendar time, for young children participating in WIC.

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1. Introduction

A central issue of food assistance program administration is: Which certification period is "best?" There are arguments both for longer certification durations and for shorter. Differences in policy recommendations may reflect, in part, fundamental differences of what "best" means and how to reconcile competing program goals. Policymakers and program managers face challenges in balancing the tradeoffs between benefit targeting, administrative cost and client access. Each of those program goals is affected by the length of the certification duration (CD) for a program's persons or households (hereafter, "clients").

A variety of economic factors affect CDs in principle and in practice. Income volatility is central among them. The rate at which a client transitions from eligibility to ineligibility due to fluctuation in income is a key parameter of interest, affecting program management, operations and targeting. Recertification is an administrative tool by which the program agency can ascertain whether the client has become ineligible and terminate the client's program benefits.

This paper develops an economic model of CDs designed to examine a wide array policy factors, including the roles of income volatility, the dollar amount of program benefits, recertification costs for the program agency and for the client, the excess burden of taxes, and the social discount rate. The roles of these factors are considered from five different "perspectives" that value differently the effects of changes in CDs: a Client Perspective, a Taxpayer Perspective, a Social Perspective, a Government Budget Perspective, and a Program Manager perspective. After developing the economic model, the paper uses it to simulate a set of optimal CDs for young children in the Special Supplement Nutrition Program for Women, Infants, and Children (WIC). The actual CD for WIC children is 6 months. In a baseline simulation, the optimal CD for both the Taxpayer Perspective and the Government Budget Perspectives is 6 months. The optimal CD from the Social Perspective, which treats the dollar value of the WIC food package as a transfer payment, is 12 months. Alternative parameter values are assigned and simulations re-run for sensitivity analysis.

[1] The views expressed in this paper are those of the author and not necessarily those of ERS or USDA.

The literatures on the dynamics of income, poverty status, and food assistance program participation have results that are informative about the life-course and income trajectories of low-income people. Annual longitudinal data, such as the Panel Study on Income Dynamics (PSID), sustain important research that adds to our understanding of these long-term processes, and the extent to which people participate in government programs during difficult times. However, the certification periods of food assistance programs are measured in terms of months, not years. Studies that examine behaviors and outcomes at the monthly level are especially helpful for certification policy. Such studies may make use administrative data, specialized data-collection efforts (perhaps associated with experimental studies), and especially the Survey of Income and Program Participation (SIPP). Recent contributions include Blank and Ruggles (1995), Farrell, Fishman, Langley, and Stapleton (2003), Cody, Gleason, Schechter, Satake, and Sykes (2005), Newman (2006), and Ribar, Edelhoch and Liu (2006a, 2006b). Some research has considered income volatility specifically as it affects WIC eligibility measurement, in which the use of monthly income or annual income is a central issue. Examples that are reviewed here are Gordon, Lewis and Radbill (1997), Ver Ploeg and Betson (2003), and Bitler Currie and Scholz (2003) whose study specifically addressed the role of CDs in WIC on eligibility measurement. Two works that examine the effects of CDs in the FSP are by Kabbani and Wilde (2003), who use an econometric approach to investigate effects associated with "short" certification periods of 1 to 3 months, and by MaCurdy and Maruffo (2006), who develop a full-scale SIPP-based simulation model to examine monthly income dynamics and issues associated with the longitudinal experiences of households participating in the FSP. This paper is similar in intent to Kabbani and Wilde and to MaCurdy and Maruffo in that the effects of CDs are a focus, but the paper's model is applied to the WIC program.

While the paper's CD model is part of a literature on income volatility and food assistance programs, the formal structure of the CD model most closely resembles so-called "inspection" or "preparedness" models for maintenance and replacement of stochastically failing equipment. In these models, equipment that is initially "good" at installation can "fail" at any time, and the current state of equipment—good or failed—is uncertain unless and until the equipment is inspected. These preparedness (or inspection) models include work by Radner and Jorgenson (1962) and Jorgenson, McCall, and Radner (1967). The paper reviews in detail a model from Barlow and Proschan (1996). The decision-making environment of these models matches the information environment of the CD model: a program agency is uncertain about the client's current state—eligible or ineligible—unless and until the client is recertified. The paper's simulations most closely resemble work in Greenfield and Persselin (2002).

Warranted and Unwarranted Benefits. The U.S. Department of Agriculture (USDA) administers 15 domestic food assistance programs designed to provide access to food to low-income people and to children. Major programs include WIC, the Food Stamp Program (FSP), and school meals programs—the National School Lunch and School Breakfast Programs (NSLP, SBP). Policymakers set criteria for each program that determine which clients are eligible to receive program benefits. A common criterion is a limit on a client's income relative to the poverty line, a figure that takes into account both the household's money income and household composition. Income limits are 185 percent of poverty for WIC and for reduced-price school meals, and 130 percent of poverty for the FSP and for free school meals.[2]

A program agency evaluates the application of a client and identifies if the client is eligible.[3] An eligible client is "certified" as such and the program determines the amount of benefits.[4] The agency also identifies the *certification period*—the length of time during which the client may receive benefits before returning to the office for recertification, a process that is nearly as thorough and time-consuming as the initial application. The client will no longer receive benefits unless the agency recertifies, based on *current* information, that the client is *currently* eligible.[5] The agency terminates the benefits of a client who is detected to be ineligible at recertification (or who does not appear for recertification). A client who is found to be eligible at recertification continues to receive benefits until the next recertification.

In the CD model, the program benefits a client receives in a given time period are divided between two categories—warranted and unwarranted. This division is simple in the WIC program, which typically treats a month as the relevant unit of time (an important point examined in the paper). Program benefits in WIC are *warranted* if the client is eligible in the same month that the client receives the program benefits, i.e., if the client's current monthly income meets the monthly eligibility guidelines. If instead the client is ineligible in the current month, due to income volatility since the application, the client's

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[2] Income is not the only criterion for these programs. For example, the FSP has upper limits on assets, among other criteria. To be eligible for WIC, a client must be considered by a health professional to be at nutritional risk.
[3] An increase in the depth, extensiveness or thoroughness of the application or recertification can be expected to result in a more accurate assessment by the program of the client's "true," but unobserved, eligibility status. However, the paper does not examine issues of supplementing self-reported information by documentation or verification. The paper simply presumes "true" eligibility status is identifiable by the agency in order to focus on the timing, as opposed to the thoroughness, of recertification.
[4] While benefits for WIC clients are independent of income, school meals benefits vary across the three meal categories (free, reduced-price, and full-price) and FSP benefits can vary dollar-for-dollar with income.
[5] For simplicity, this paper will use the terms "eligible" and "ineligible" to refer to meeting income guidelines in the *current* time period. Some reports may use the term "eligible" to be synonymous with "certified," but that usage becomes confusing in this paper which seeks to distinguish between being *eligible* (in the current month according to the income guidelines) and being *certified* by the program office to receive benefits in the current month.

benefits in that month are said to be *unwarranted*. Unless the client's monthly income drops back below the income guidelines before the next recertification, WIC will terminate the client's benefits at the next recertification once those benefits are detected to be unwarranted. Terminating unwarranted benefits fosters targeting of benefits to those clients who are eligible, promoting program integrity, and it may result in a saving of taxpayer dollars.[6]

For the FSP, distinguishing warranted from unwarranted benefits is slightly more involved than for WIC. In WIC, benefits in the current time period are either all warranted or all unwarranted. In the FSP, benefits may be both partly warranted and partly unwarranted. FSP benefits depend on the client's income, with higher-income clients receiving lower benefits.[7] If a client's income increased by a large enough amount (from the time of initial application), the client is *currently* due a smaller amount of warranted benefits than the amount of benefits the client is actually receiving (an amount set at application). In these cases, the amount of unwarranted benefits is the difference between current benefits received and (current) warranted benefits.[8]

Certification Periods in Practice. Certification periods vary across food assistance programs, as well as across participants and subgroups of participants within a program. Some food assistance clients have a CD as short as 1 month, while others have a CD as long as 24 months.

In WIC, different categories of clients—infant, child, pregnant woman, or breastfeeding or nonbreastfeeding postpartum woman—do not all share a common certification period. The certification period for WIC children (ages 1 through 4 years) is six months. In the school meals programs, children receiving free or reduced-price school meals are certified through one month after the start of the following school-year, creating for most children a certification period of approximately one year.[9]

In the FSP, federal regulations require that non-elderly non-disabled participants be certified at least once per year and States have flexibility for recertifying more frequently. Elderly or disabled clients, whose incomes tend to be relatively stable, may be certified once every two years. In the FSP, the average

[6] If all clients could be counted upon to self-report any changes in their eligibility conditions to the program agency, no unwarranted benefits would ever need be paid and no recertification need ever be done. Throughout the paper it is assumed that clients do not self-report such changes: a recertification is required to ascertain eligibility.
[7] An income-dependent benefit schedule is used to target benefits more fully to those clients with relatively less income.
[8] If income should drop within the certification period, the client can notify the local FSP office to be certified right away for the larger amount of warranted benefits for which the client is now eligible.

length of the certification period for all FSP households (except households receiving the minimum benefit) was 10 months in fiscal 2004. Table 1 shows the distribution of lengths of certification across FSP households. A small percentage of clients have a one-month long CD. The table also shows the distributions for two subgroups based on whether the household had earnings, and whether the household had an elderly member. Households with earnings tend to have shorter certification periods—averaging 8 months—than FSP households overall, consistent with efforts by FSP program agencies to detect unwarranted benefits due to volatility in earnings (which tend to be higher than volatility in other sources of income). The average CD for households with elderly members is 16 months, reflecting the longer time limits allowed by federal regulations. The spikes in the distribution of certification at 6 months and 12 months show the prevalences of semi-annual and annual CDs.

The FSP gathers information from clients more frequently than at intermittent recertifications, using the tools of monthly reporting and change reporting. It is likely to be cost-effective for a program to use a "mix" of information-gathering tools. However, the CD model abstracts from differences between the modes and detail of various tools. By ignoring differences among types of reports and program-client interactions, the CD model in effect treats any occasion of program-client interaction to be a "recertification" at which updated information is acquired.

Certification Periods in Principle. It may be appropriate, in principle, to conduct recertifications more frequently for a program, or for a subgroup of clients within a program, if client incomes are highly volatile. The report *Characteristics of Food Stamp Households: Fiscal Year 2004* states "The certification period varies [in the FSP] according to the likelihood of a change in a food stamp household's financial circumstances." (p. 7) Frequent recertification has the advantage of detecting unwarranted benefits more quickly and discontinuing such payments. It can be also expected that, in principle, the size of unwarranted benefits—not just the likelihood—would matter for determining CDs: $300 of unwarranted benefits would matter more than $30 worth.

Targeting benefits to those who are currently eligible is not the only goal of program administration. Each recertification is a burden on the client, entailing monetary and time costs and possibly psychic costs. Frequent recertifications may act as a barrier or disincentive to program participation, potentially decreasing participation by the very clients for whom the program was established to support. Moreover, recertification involves an administrative burden that requires economic resources in terms of program

[9] Children who apply for free or reduced-price school meals during the school year, rather than at the outset of the school year, have the same recertification date as other children and therefore have a shorter certification period.

staff time, equipment and material. Information from the client must be processed to ascertain whether the client is currently eligible, and records must be updated accordingly. If recertifications are conducted infrequently instead of frequently, then client access is promoted and administrative costs are saved.

To identify a "best" certification policy, the CD model uses a particular cost-minimization criterion that, like any criterion, necessarily balances competing program goals its own particular way. The criterion turns out to be extremely flexible, permitting simulation of different optimal CDs based on different perspectives (discussed in section 2). Nevertheless, other criteria may be imagined, reflecting different notions of "best" and of how to balance in principle among competing program goals.

An important feature of the CD model is that it considers in detail the tradeoffs between program integrity and recertification costs only. An important tradeoff that is not fully incorporated in this paper's model, in order to concentrate on just one pair of goals, is the issue of client access. The CD model presumes that variation in CDs does not affect a client's willingness to participate in the program.[10] As a result, the length of the model's "optimal" CD is likely to be *shorter* than would be the case in a fuller model that explicitly includes client access. If others aspects of the CD models are reasonable, the CDs examined in this paper can be interpreted as lower bounds. As scarce as research results are on CDs for food assistance programs, a reasonable lower bound is in itself informative.

Descriptive or Prescriptive? There is a tension between positive and normative modeling of government decision-making. To date, policymakers have set certification periods without the benefit of a formal model. Should the CD model be used *descriptively*, as a mathematical formulation of the factors that influence actual decisions on CDs? In this interpretation, the model's usefulness would be judged by its ability to account for the CD outcomes of the actual policymaking process—just as a model of firm behavior or of a market is judged by its ability to explain for observed data. Discrepancies between the results of the model and actual practice are attributable to misspecification of the model, to its limitation as an imperfect, incomplete description of the behavior of policymakers and program managers.

Alternatively, the CD model could be used *prescriptively*, as a normative guideline of how policymakers and program managers should be setting CDs to balance better among the program goals. If the CD model is interpreted as a normative model, discrepancies between the model and existing policies may attributable to policymakers having incomplete information about program conditions and client

[10] This omission may be especially pertinent for working-poor households for whom the time and hassle costs of arranging work schedules to attend a recertification in person may be relatively high.

parameters (such as the degree of income volatility). In this interpretation, discrepancies may be considered in light of adjusting the policies to the model rather than the other way around. Viewed in this fashion, the model is a contribution to the policymaker's toolkit.

2. A Steady-State Model and Valuation Issues

Under what conditions does conducting a recertification for a given client pass a benefit-cost test? To answer this question, highlight issues of economic valuation and explore the difference between positive and normative analysis, this section develops a simplified steady-state model of recertification.

Steady-State Model. In the steady-state model, time is treated as discrete and the relevant time period is a month. Clients are homogeneous in two respects. First, each client receives the same monthly program benefits of M (which are paid at the start of the month). Second, a Bernoulli assumption is adopted that the event "currently ineligible" occurs with probability P for any given client in any given month; while each client was eligible at application, a long time has now passed to reach a steady-state in which each client shares the same ineligibility probability P. From these two assumptions, program benefits for any given client can be divided between unwarranted and warranted benefits which, in expectation, are PM and (1-P)M respectively. Let C_A and C_C represent recertification costs paid by the agency and the client. The excess burden (or marginal efficiency cost) of taxes, by which program benefits and administrative costs are financed, is given by ε (per dollar of taxes). The social discount rate is r per month.

Table 2 shows discounted values for conducting a benefit-cost analysis that compares a policy of "Never Recertify" and a policy of "Recertify Once Now." The table shows three different perspectives: a Taxpayer Perspective, a Client Perspective, and a Social Perspective that adds up all benefits and costs whether they fall to a taxpayer or a client.[11] Two other perspectives, introduced below, are a Government Budget Perspective and a Program Manager Perspective.

For the policy of "Never Recertify," there are no recertification costs (from any perspective) in row (1). Program Benefits in row (2) take on three values. The discounted value from the Client Perspective of M paid monthly is [(1+r)/r]M— the sum of this month's payment of M plus the discounted value of M paid monthly in the future of M/r. This value is a "benefit" for the client, indicated by the positive sign "+" in the Client Perspective column. From a Taxpayer Perspective, program benefits constitute a "cost,"

[11] This distribution-based approach of supplementing the Social Perspective of standard benefit-cost analysis with the Taxpayer and Client Perspectives is borrowed from the literature on benefit-cost analysis of job training programs, e.g., Mallar, Kerachsky, Thornton, and Long (1982).

resulting in the "-" sign, and each dollar entails an excess burden of ε, resulting in the factor (1 + ε). Inclusive of efficiency losses due to taxation, the taxpayer bears a larger burden than the client actually receives in program benefits. The Social Perspective column shows that only the discounted value of -εM remains after taking into account the transfer-payment aspect of program benefits.[12]

Rows (4) through (7) show the consequences of conducting a recertification once at the start of the current month, the instant before program benefits are paid out. Recertification costs are shown in row (4). The next rows show possible outcomes of recertification. In row (5), the client is currently ineligible (with probability P) and program benefits are terminated. With probability (1-P) the client is currently eligible, and program benefits are continued forever in row (6) (as in row (1)). The Expected Net Present Values in row (7) shows the expected values associated with conducting a (single) recertification as the sums of (4) and the probability-weighted values in (5) and (6).

It is helpful to divide the values of row (6) into the portions expected to be unwarranted and warranted. The client is eligible this month—as determined by the recertification—but by the Bernoulli assumption the client has probability P of being ineligible *next* month and every month thereafter. In expectation, discounted unwarranted benefits are PM/r, as shown in the Client Perspective column. Discounted warranted benefits has two components: the immediate payment M and the (expected) discounted warranted benefits of (1-P)M/r. The unwarratend and warranted figures in the Taxpayer and Social perspectives take into account the factors (1+ε) and ε, as appropriate.

The net gains of adopting the Recertify Once Now policy in place of the Never Recertify policy are the differences between row (7) and row (3). These Expected Net Present Values are given in row (8). For all parameter values, recertification is strictly a losing proposition from a Client Perspective : not only does the client bear cost C_C, there is a chance (when ineligible) of losing program benefits altogether. Turning to the Taxpayer and Social perspectives, the expected discounted value of recertification (once) can be written as:

$$(1.1)\ E[NPV] = k\left(\frac{1+r}{r}\right)PM - C$$

[12] This approach differs from that found in the analysis of the Job Corps found in Mallar et al. (p. 9) due to the inclusion of excess burden. Mallar et al., treat changes in public transfers as zero-sum transfer payments. If a Social Perspective is adopted and excess burden is not recognized, then conducting a recertification will *never* pass a benefit-cost analysis: recertification uses up economic resources and only affects the distribution of a zero-sum transfer payment, resulting in a cost-effective policy of Never Recertify.

where *k* takes on the value of either (1+ε) or ε from a Taxpayer or a Social Perspective, in turn, and C equals either $(1+\varepsilon)C_A$ or $[(1+\varepsilon)C_A + C_C]$ from the two perspectives. The policy of conducting a recertification passes a benefit-cost analysis when the E[NPV] in (1.1) is positive.

As simple as it is, an interesting result emerges from (1.1). The term PM in (1.1) constitutes *unwarranted* benefits even though rows (3) and (6) involved M—monthly benefits, whether warranted or unwarranted. The term P enters the expected value of conducting a recertification through the probability-weighting of the events of "eligibility" and "ineligibility" in rows (5) and (6) does. In row (8) (or in (1.1)), what a Taxpayer or a Social Perspective each weigh is whether discounted *unwarranted* benefits exceed recertification costs (adjusting for excess burden). These two perspectives ignore warranted benefits—which get paid to the eligible client *whether or not* there is a recertification—when considering the "bottom line" in (8). Only unwarranted benefits are curtailed by recertification.

A recertification tends to pass a benefit-cost test when: income volatility is relatively high (in the sense of a high value for P), monthly program benefits are relatively high, the excess burden of taxation is relatively high, the discount rate is relatively low, and the recertification cost is relatively low. In the CD model below, similar results will be established between these various factors and the timing, rather than the existence, of recertification.

Treating ε as zero in the Taxpayer Perspective provides a special case that is called a Government Budget Perspective. When ε is zero, E[NPV] in (1.1) reduces to a solution to a budget-minimization problem. If a program agency were to adopt a perspective of save-the-taxpayer's-bookkeeping-dollars by minimizing program expenditures—subject to the constraint of fully and faithfully paying program benefits to eligible clients—then the only alterable program expenditures belong to one of two categories: unwarranted benefits and (agency) recertification costs. The policy decision embodied in (1.1), when ε is zero, constitutes minimizing program expenditures by weighing the expected present value of (saved) unwarranted benefits against the (current) recertification cost.

While the Government Budget Perspective can be obtained from a Taxpayer Perspective, it is worth recognizing it as an important perspective in its own right. Policy discussion is sometimes framed with reference to the on-the-books government budget. For example, in an examination of the literature on the "effectiveness" of work-related training programs for welfare recipients, the Congressional Budget Office (CBO) devoted a section to "Effect on Government Budgets," recognizing that such programs can reduce transfer payments and thereby save taxpayer funding (U.S. Congressional Budget Office, 1987). Another

CBO analysis on work-welfare programs noted that for food stamps recipients who do not receive cash welfare, turnover is high and "[work] approaches that can be provided quickly, with immediate impact and low costs, would be more likely to result in *net budget savings*. More costly and intensive services, such as skills training or subsidized employment, would be less cost effective because recipients leave the food stamp program rapidly and *reduced benefits* would not justify large expenditures for such efforts." (U.S. Congressional Budget Office, 1983, p. 28, emphasis added)

The Government Budget Perspective is one that will be featured in the CD models, with excess burden re-introduced after the models' primary results are derived.

Valuation Issues. Table 1 already illustrates one major theme concerning valuation by presenting three different perspectives for evaluating recertification policy. The remainder of this section considers other valuation issues.[13]

It should be recognized that Table 1 does not take into account whatever value *taxpayers* receive from providing program benefits (at least to clients who are eligible). A consumption-externality argument states that (at least some) taxpayers derive utility when low-income households consume food, and the value to the taxpayer of that activity may exceed the program's dollar cost (inclusive of excess burden). The table does not incorporate that possibility. Neither does the table recognize that the value of program benefits to the *client* may differ from the value of program benefits measured in dollar terms. Food assistance is an in-kind support program. A client may value a dollar of food assistance differently (and presumably less than) one dollar.[14]

It may be appropriate for table 1 to ignore these two valuation issues for the recertification problem at hand. The issue here is not a full-fledged program evaluation of the existence (vs. the absence) of a food assistance program. A program evaluation of that sort would consider and appropriately value all the program's features, including how it addresses externalities accruing to taxpayers and the form of the benefit (in-kind versus monetary support). Instead, the policy issue at hand is setting the CD, a policy

[13] A similar set of perspectives for benefit-cost analysis of child welfare program is provided in Foster and Holden (2004), who consider the perspective of the family, the state agency, other members of society (taxpayers, victims of crime, and private or community services), and society as whole.

[14] It is possible in a household-bargaining framework that a dollar of in-kind aid may be valued by one member of the household—but not the other—at greater than a dollar. Perhaps one spouse strongly prefers in-kind aid over monetary aid because the other spouse may use monetary aid for goods considered detrimental (or at least not beneficial). The test of such an argument is whether the spouse would be willing to give up some amount of in-kind aid for the aid to remain in-kind rather than being converted to monetary aid.

that takes for granted the existence, desirability, and form of the program. The relevant benefits and costs associated with a recertification policy are posited here to take the form of unwarranted benefits and of recertification costs paid by the agency and the client.[15]

As noted earlier, a *prescriptive* approach to policy would interpret the CD model (and this section's steady-state benefit-cost analysis) as identifying what policymakers and program managers should be considering when adopting a Taxpayer or a Social Perspective for setting CDs. Alternatively, the CD model could be considered a *descriptive* model of policy decisions, hypothesizing that policymakers and program managers behave "as if" they have adopted one of the perspectives considered. A third public-choice approach to the descriptive modeling of policy decisions is outlined next. In it, unwarranted benefits and recertification costs are examined to see how they each affect *outcomes for the decision-maker*, who is called here the Program Manager.

The Program Manager perspective may not match any of the perspectives presented already. The Program Manager may not necessarily act as an "agent" for the Taxpayer, or follow the guidance of those who urge benefit-cost analysis from a Social Perspective. After all, food assistance program benefits—whether warranted or unwarranted—are financed entirely by USDA. Does the Program Manager really give the same weight to those dollars as to State and local dollars? A public-choice approach would argue that the Program Manager chooses a CD policy based on: (a) the probability that federal or State oversight activities detect the presence and amount of unwarranted benefits, (b) the probability that penalties will be imposed in terms of the Program Manager's salary or promotion possibility, and (c) the size of those penalties. Similar arguments can be made concerning administrative costs of recertification. The extent to which administrative costs are shared by USDA and the State or local program agency can depend on whether FSP, school meals, or WIC is considered. Does a Program Manager give full weight to all recertification costs even if a different level of government bears a portion of those costs? What valuation does the Program Manager place on recertification costs that are paid by the client? Finally, it is possible the Program Manager construes program benefits to be a "benefit"—either because they make the agency budget bigger or because the Program Manager is genuinely interested in providing program benefits to participants. If so, the Program Manager's decisions may reflect, at least in part, a Client Perspective's treatment of program benefits.[16] Thus, from a public-choice approach, it is not clear that

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[15] Fuller consideration of the consumption-externality issue would be important for an expanded analysis of recertification that formally models the client access issue.

[16] Boardman, Greenberg, Vining, and Weimer (1996) contrast the perspectives of "Spenders" and "Guardians." The Spender adopts a constituent's point of view in interpreting program expenditures as "benefits." The budgetary focus of the Guardian means that costs that don't appear in a program budget, such as the recertification cost paid by

the benefit-cost analysis as presented in Table 1 captures all the relevant aspects of decision-making for a descriptive model.

Developing a full public-choice Program Manager perspective is beyond the scope of this paper, but it seems its major (perhaps only) issue concerns "weights" for various dollars. In a public-choice version of the CD model, unwarranted benefits and recertification costs each matter to the Program Manager. The issue is what (separate) weights to attached to those dollars to reflect the incentives on the Program Manager. The structure of the public-choice version of the CD model, and the underlying nature of the tradeoff between unwarranted benefits and recertification costs, would be much the same as this paper's model. The weights would no doubt affect the model's solution of an optimal CD, but the public-choice theorist would still turn to the CD model's apparatus for a solution.

3. Barlow-Proschan Inspection Model

Because the central question of the CD model involves time—specifically, the duration for the certification period—the CD model has similarities to other economic models that also involve time. As noted, the CD model's formal structure most closely resembles "inspection" models. Thus, it is helpful to review a simple inspection model taken from Barlow and Proschan (1996; pp. 107-115). The B-P model contains many elements that will carry over to the CD model's problem of recertification. The B-P model was originally developed in the context of inspecting of durable equipment. The decision is when to inspect equipment to discover whether it is "good" or "failed." The discussion here of the B-P model translates the model's language to the food assistance program environment.

Structure of the B-P Model. At the time of application an initial certification, time 0, the agency knows that the client is eligible. At time following application, a client is in one of two states: eligible or ineligible. The client's state is unknown unless a recertification is conducted. The client's time of "failure"—of transition from eligibility to ineligibility—is a random variable. Although the client's state is unknown, the program agency knows the failure distribution by which a client transitions from eligibility to ineligibility. In the B-P model, ineligibility is an "absorbing" state: once the client transitions from eligibility to ineligibility, the client remains ineligible forever. Once ineligibility is detected, further possible recertifications are cancelled and the problem is over. However, the CD model of the next section allows for the complication of transitions between both states—from eligibility to ineligibility and back again.

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the client, are ignored. The so-called "Analyst" perspective, which is the Social Perspective in this paper, is

The failure distribution F(t) is the cumulative probability distribution showing the probability that the time to "failure"—ineligibility—is less than or equal to *t*. Although the CD model makes use of exponential distributions, for now F(t) is treated as a continuous cumulative probability distribution. Let f(t) be the associated probability density for F(t) given by dF(t)/dt, or

$$(3.1)\ F(t) = \int_0^t f(u)du = Prob\{time\ to\ failure \leq t\}$$

A companion to the failure distribution is the reliability (or "survivor") function R(t), which is the probability a client is still eligible as of time t, or equivalently, the probability that the time of transition to ineligibility is greater than t. The relationship between R(t) and F(t) is simply

$$(3.2)\ R(t) = 1 - F(t) = Prob\{time\ to\ failure > t\}$$

The hazard rate (also known as the "hazard function" or the "failure rate") h(t) is the probability that the client will transition to ineligibility within an instantaneously small interval at time t, conditioned on having reached *t* as eligible (i.e., conditioned on not transitioning prior to time *t*). The hazard rate can be expressed in terms of the failure distribution or the reliability function:

$$(3.3)\ h(t) = \frac{F'(t)}{1 - F(t)} = -\frac{R'(t)}{R(t)}$$

The B-P model balances two types of costs: total recertification cost and an "undetected-failure" cost that grows linearly with the time that passes from transition to ineligibility and recertification.[17] Total recertification cost equals a per-recertification cost *c* times the number of inspections. The undetected-failure cost is the product of a flow cost, *m*, per instant of (continuous) time and the length of time between transition and detection of ineligibility. In this paper, *m* represents a flow of program benefits, in which case "undetected-failure" cost represents (cumulative non-discounted) unwarranted benefits.

More frequent recertifications raise total recertification cost, but lower unwarranted because the transition to ineligibility—if and when it happens—will be detected more quickly with more frequent recertifications. The criterion in the B-P model is expected (total) cost, given by

associated by Boardman et al. with standard benefit-cost analysis.

[17] Undetected-failure cost would have different interpretations depending on the application of the model. B-P suggested examples of a missile, a drug stored for epidemics, and a machine producing output. An undetected failure would be: a non-functioning missile, which has costs in terms of effectiveness of national defense; an impotent drug, which has costs in terms of preparedness for an epidemic; or a defective machine, which produces output that doesn't meet quality specifications.

$$(3.4)\quad E[C]=\sum_{k=0}^{\infty}\int_{x_k}^{x_{k+1}}\left[c(k+1)+m(x_{k+1}-t)\right]dF(t)$$

where $x_1 < x_2 < x_3 \ldots$ are the successive times at which recertifications occur, given $x_0 = 0$. B-P note that if F(t) is continuous with finite mean, then there exists a sequence of recertification times that minimizes E[C].[18] The expected cost criterion of (3.4) includes (non-discounted) program expenditures that would register most directly in a Government Budget Perspective.

B-P consider a special case in which failure occurs with certainty by time T, i.e., F(t) = 1 for t ≥ T. This case is labeled here as a "finite-horizon" model. B-P identify that if but one inspection is done at T, then

$$(3.5)\quad E[C]=c+m\int_0^T (T-t)\,dF(t)=c+m\int_0^T F(t)dt$$

gives the associated expected cost, which consists of a single per-recertification cost *c* (paid with certainty) plus the unwarranted benefits.[19] In (3.5), if the client becomes ineligible at the very first instant 0, unwarranted benefits would simply equals *m*T. However, for *t* prior to T there is only a probability, given by F(t) < 1, that the unit was failed as of *t*: multiplying *m* by the sum (integral) of those probabilities across the [0, T] interval gives the expected unwarranted benefits in (3.5).[20] B-P note that if an earlier inspection is conducted at time x < T, in addition to the inspection at T, then

$$(3.6)\quad E[C]=\int_0^x \left[c+m(x-t)\right]dF(t)+\int_x^T \left[2c+m(T-t)\right]dF(t)$$
$$=c+[1-F(x)]\,c+\left\{m\int_0^T F(t)dt-F(x)\,m(T-x)\right\}$$

[18] In contrast to the expected cost criterion of the B-P model in (2.4), the preparedness model of Jorgenson et al. considered criterion involving average cost per unit of "good" time, taking into account "down" time during which equipment is not in service due to repair and replacement.

[19] The specification for unwarranted benefits presumes that the client receives program benefits, following a transition to ineligibility, until recertification. If instead the client were to leave program participation prior to recertification, unwarranted benefits would be lower. Thus, the model assumes that participation does not end between recertifications. Evidence in support of this assumption for the FSP is in Riber et al. (2006a, 2006b), who use a fine-resolution of 6-day periods that the FSP participation hazard rates for periods between recertifications are relatively low, i.e., "spikes" occur at recertification. Evidence in support of this assumption from WIC is that 92 percent of those who are enrolled in WIC pick up the vouchers by which they obtain WIC-approved foods at retailers (U.S. Department of Agriculture, 2002). If inter-recertification exit from participation were deemed serious, the model could be adjusted so that F(t) would measure income eligibility transition among continuing participants instead of all applicants.

[20] It is noted for completeness that recertification reveals whether the client is eligible or ineligible at that moment, but not the past time of transition. The amount of unwarranted benefits can not be determined, even in retrospect.

In (3.6), the first term *c* is the inspection that occurs with *certainty* at some time x (not yet identified). The second term indicates the *expected* cost of recertification at T. The recertification at T does not occur with certainty in the two-inspection regime: recertification at T will occur if the client is still eligible at x—an event with probability 1 – F(x). The final two terms in brackets are related. The first term is, again, the expected unwarranted benefits for a single recertification (at T), while the second term is a deduction from that total. The deduction is a gain, in expectation, from conducting an extra, earlier recertification at x and finding the client is *already* ineligible. In that event, the flow of unwarranted benefits is stopped, avoiding the continued flow of such payments up to T. The avoided payments equal *m*(T – x), which are a gain that is obtained with probability F(x). The two terms in brackets constitute the (net) expected unwarranted benefits for a policy regime of two inspections.

It will be helpful below to label the first two terms in (3.6) as $C_R(x)$—the (expected) cost component associated with recertification—and to label the two terms inside the brackets as $C_U(x)$—the (expected) cost component associated with the (net) unwarranted benefits.

Here, the difference between (3.5) and (3.6) is interpreted as the expected saving, E[S(x)], from adding an inspection at x < T, given by:[21]

$$(3.7)\ E[S(x)] = F(x)m(T-x) - [1-F(x)]c$$

Maximization of expected saving in (3.7) with respect to x yields an optimal x* that is the same recertification time obtained from (3.6) from minimization of expected cost.

Solution of the B-P Model. The optimal inspection is found where the slopes of the two functions $C_R(x)$ and $C_U(x)$ are equal (in absolute value), as shown in the first-order condition that results from minimization of (3.6):

$$(3.8)\quad \frac{dE[C(x^*)]}{dx} = 0 \rightarrow \frac{dC_R(x^*)}{dx} = -\frac{dC_U(x^*)}{dx}$$

[21] B-P use the difference between (3.5) and (3.6) to consider conditions on F(t) and model parameters under which E[C] is lower when there is a single inspection at T or, conversely, when one or more additional inspections occur prior to T. It will be presumed here that conditions are met by which it is optimal to have a second inspection at x, and policy regimes with more than two inspections are not considered in this review of the B-P model.

Inasmuch as the slope of $C_R(x)$ is negative throughout (given by –F'(x)c), optimal inspection x* is found where a marginal *increase* in $C_U(x)$ is just balanced by the *reduction* in $C_R(x)$. The slopes $C_R(x)$ and $C_U(x)$ are composed of three terms, and first-order condition can be re-written as:

$$(3.9)\ F'(x^*)c + F'(x^*)m(T - x^*) = F(x^*)m$$

A marginal increase in x (i.e., a slight delay in the time of recertification) raises the probability of ineligibility—and thus the probability of *detection* of ineligibility at recertification—by F'(x). Perhaps counter-intuitively, delayed recertification is actually desirable (up to some point) not "in spite" of the prospect of an increased chance of ineligibility but, rather, "because" of it: if ineligibility is detected at x, then the second inspection at T is cancelled, resulting in a gain of *c*. That is, if F'(x) is large in the region, then, at the marginal instant before x, there is a big gain to be had by holding off the recertification another moment. The motivation for waiting is to try to detect client ineligibility just *after* its (much more likely) occurrence than before. Thus, a *reduction* in a cost along the $C_R(x)$ curve of—the term F'(x)c in (3.9) is, in absolute value, the slope of $C_R(x)$ in (3.8)—is interpretable as a "marginal benefit" or "marginal gain" from a slight delay in recertification.

The slope of $C_U(x)$ has two terms that are separated and expressed on the two sides of (3.9) The second term on the left-hand side is one of these terms. It shows that unwarranted benefits of *m*(T-x) are saved if ineligibility is detected. Again, F'(x) plays a role. Together, the two incremental gains on the left-hand side of (3.8) represent the overall expected "marginal gain" from a delay in x.

On the other hand, if (a) the client is already ineligible, and (b) the recertification is delayed one instant, then an unwarranted benefit of *m* is paid (for that short interval). The client is already ineligibile at x with probability F(x), making F(x)*m* the expected "marginal loss" from delay. As (3.9) shows expected total costs are minimized by an inspection at x* that balances expected marginal "gains" and "losses."[22]

To isolate the characteristics of the failure distribution F(t), the (3.9) could be re-expressed as

$$(3.10)\ \frac{F'(x^*)}{F(x^*)} = \frac{m}{c + m(T - x^*)}$$

[22] Because the slope of $C_U(x)$ involves two terms that are put on separate sides of (3.9), the "marginal gain and loss" interpretation does not correspond one-for-one to the two component cost curves $C_R(x)$ and $C_U(x)$ or their slopes.

where F'(x)/F(x) is the instantaneous percentage change in F(x); it is noted that the term is not the hazard rate of (3.3). The expression [c + m(T-x)] represents the amount of cost saved at recertification (equaling the cost of the second recertification at T plus unwarranted benefits between x and T) conditional on detecting ineligibility at x. The right-hand side of (3.11) its instantaneous percentage.

4. The CD Model

While the B-P model just reviewed is a simple version of their finite-horizon model, B-P also provided an infinite-horizon model. The CD model developed next adapts their infinite-horizon model with three changes. First, the CD model introduces discounting to take into account the time value of money.[23] Thus, the new criterion of the CD model will be expected discounted cost E[DC], rather than the criterion of expected cost E[C]used in the B-P model. Second, the CD model uses not only a failure distribution for transitions between eligibility (state E) and ineligibility (state I), but a second "failure" distribution for transitions back from I to E. This complication permits relaxing the assumption of the B-P model that ineligibility is an absorbing state. A third difference is that B-P derive some results for general failure distribution functions, and treated examples using exponential distributions as special cases. The CD model, in contrast, examines the problem using exponential functions alone, which permit the use of dynamic programming techniques to derive the model's solution. It is convenient to continue to derive results from the Government Budget Perspective.

Stochastic Process with Two-State Transitions. The two-state transition process more usually goes by the label "alternating renewal process." Lancaster (1990) considered an alternating renewal process for the states of employment and unemployment. Heyman and Sobel (1982) examine the process as an example of a "birth-and-death" process. Boskin and Nold (1975) considered such a process when examining the states of "participation" and "non-participation" in modeling the AFDC program. The CD model simply uses the same stochastic process to model eligibility-ineligibility transitions.

The CD model adopts the common simplifying assumption that the reliability function (or survival function) R(t), for the eligible client to remain eligible, is exponential given by

$$(4.1)\ R(t) = e^{-\lambda t}\ ,\ t \geq 0$$

The hazard rate h(t) of (3.3) for transition from E to I is a constant given by λ. The failure distribution is

[23] While B-P are aware of discounting, and cite works that use discounting, a criterion often analyzed by B-P, by engineers and by applied mathematicians is non-discounted expected cost or expected cost per unit of time (over an infinite time span), which can be described as a criterion that is reached in the limit (in multiple-cycle models) as the discount rate approaches zero.

$$(4.2)\ F(t) = 1 - e^{-\lambda t}\,, \quad t \geq 0$$

The expected duration of a *continuous* spell in E is (1/λ) from the start of the spell.

The exponential distribution has been used for equipment maintenance models by Barlow and Proschan (1996), Jorgenson et al. (1967) and Radner and Jorgenson (1962). A key feature of the exponential distribution is its Markovian or "memoryless" property. The reliability function for a client who is recertified and found to be eligibile is the same reliability function for the newly-certified entrant as of time 0. The hazard rate for a newly-certified client is 8, and at any point in the future—before or after recertification—the hazard rate is still 8. In other words, the client's stochastic time profile following recertification (given that the client is found to be eligible) is the same profile as at time zero.[24] Each recertification serves as a "regeneration" or "renewal" point that returns the problem's stochastic characteristics to time zero.

Let G(t) represent the failure distribution for transition from I to E. That is, G(t) pertains to a formerly eligible, currently ineligible client who re-enters E due to a sufficiently large income decrease, where:

$$(4.3)\ G(t) = 1 - e^{-\mu t}\,, \quad t > 0$$

A (formerly eligible) client who is ineligible at any given time exhibits the hazard rate μ for the conditional probability of transitioning from I back to E.[25] As in the case for F(t), G(t) has the Markovian property that the transition process is "memoryless." The expected duration of a *continuous* spell in I is (1/μ) from the start of the spell.

The probability that a client occupies a given state, E or I, at a given time *t* is a "state probability" given by $\pi_E(t)$ and $\pi_I(t)$, respectively. Given the means of (1/λ) and (1/μ) from the exponential distributions above, the steady-state or long-run state probabilities are:

[24] As noted by Jorgenson et al., "For the exponential distribution of time to failure . . . inspection, like replacement, serves as a point of regeneration of the investment process. This property of the exponential distribution, often referred to as the Markovian property or "lack of memory", is not shared by any other distribution of time to failure." (p. 90)

[25] The parameter μ is a transition rate of re-entry, for clients who had previously exited from eligibility, as opposed to a hazard rate of initial entry from the general population of ineligibles. Formerly eligible clients constitute a group that can be expected to exhibit (a) lower income than the general ineligible population, and (b) a higher rate at which eligibility is (re)entered.

$$(4.4)\ \lim_{t\to\infty}\pi_E(t) = \frac{\left(\frac{1}{\lambda}\right)}{\left(\frac{1}{\lambda}\right)+\left(\frac{1}{\mu}\right)} = \frac{\mu}{\lambda+\mu}$$

$$(4.5)\ \lim_{t\to\infty}\pi_I(t) = \frac{\left(\frac{1}{\mu}\right)}{\left(\frac{1}{\lambda}\right)+\left(\frac{1}{\mu}\right)} = \frac{\lambda}{\lambda+\mu}$$

As noted in Lancaster (1990, p. 97), who shows these long-run probabilities, "if state 1 durations average 4 times as long as state 2 ones then the process has probability 4/5 of being in state 1 when observed," i.e., $\pi_E(t)$ in (4.4) would be 4/(4+1).

It so happens that the steady-state probabilities are given by a state's mean spell duration relative to the sum of the two states' mean spell durations not just for exponential functions but for general functional forms for F(t) and G(t). The additional structure provided for the CD model by the use of exponential functions permits readily deriving the state probability paths outside of steady-state.

Let time 0 represent an "initial" time. Given the exponential distributions in (5.1) and (5.2), and generalizing a numerical example from Howard (1960), the state probabilities are given by:

$$(4.6)\ \pi_E(t) = \pi_E(0)\left[\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]+\pi_I(0)\left[\frac{\mu}{\lambda+\mu}-\frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]$$

$$(4.7)\ \pi_I(t) = \pi_E(0)\left[\frac{\mu}{\lambda+\mu}-\frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]+\pi_I(0)\left[\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]$$

where the probabilities $\pi_E(0)$ and $\pi_I(0)$ represent the probability that the client is in E or I at whatever "initial" time is considered. Given those "initial" values, the state probabilities in (4.6) and (4.7) each approach the same values given in (4.5) and (4.6) as *t* grows without limit.

Suppose the "initial" time of the problem is taken to be the time of application and initial certification of the client. At time of application, the program agency knows that $\pi_E(0) = 1$ and $\pi_I(0) = 0$. For the eligible client, (4.6) and (4.7) each reduce to a set of *conditional* state probabilities, i.e., to state probabilities (for E or I) conditioned on initial eligibility at time 0. This pair of conditional state probabilities can be written as $P_{EE}(t)$ and $P_{EI}(t)$ given by:

$$(4.8)\ P_{EE}(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

$$(4.9)\ P_{EI}(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

(The CD model makes thorough use of (5.7) and (5.8), and for brevity will call them "state probabilities" even though they are properly understood as conditional.) Not surprisingly, the stochastic time profiles $P_{EE}(t)$ and $P_{EI}(t)$ sum to 1 at all *t*: a client (who is eligible at time zero) is either in state E or state I at any given time.[26] These state probability paths approach steady-state values monotonically: $P_{EE}(t)$ is downward-sloping for all *t*, while $P_{EI}(t)$ is upward-sloping. What those behaviors mean, in practice, is that the probability of detecting that a client is ineligible at time *t* strictly *increases* as more time passes, a property found in the B-P model too.

Consider a recertification after T has passed since application or the last recertification. Due the "memoryless" property, the recertification serves as a "regeneration" of the two-state stochastic process of transitioning between states E and I. The stochastic time profiles $P_{EE}(t)$ and $P_{EI}(t)$ hold for an eligible client at time 0 by using $\pi_E(0) = 1$ and $\pi_I(0) = 0$ in (4.6) and (4.7). Similarly, at recertification at T, $\pi_E(T) = 1$ and $\pi_I(T) = 0$ for the client who is found, due the process of recertification itself, to be eligible at T. Using those values in (4.6) and (4.7) again results in the *same* stochastic profiles at T as were found at time 0. That is, viewed from time T, the same $P_{EE}(t)$ and $P_{EI}(t)$ time paths are obtained that were derived at initial time 0 (so long so long as *t* is interpreted as "duration time from the last recertification" rather than calendar time).[27] In short, recertification can serve as a new "initial" time.[28] The same result holds at each successive recertification, at 2T, 3T and so forth after time 0.

[26] There is another pair of probability paths (now shown) for the probabilities that the client is eligible and ineligible at time t, given that the client is ineligible at time zero; these two paths are not needed in the paper.
[27] If *t* were to represent calendar time, then notation would need to be adjusted to re-express the time as (T+t) for time following the first recertification, (2T+t) for time following the second recertification, etc.
[28] If the two failure distributions F(t) and G(t) were to exhibit non-constant hazard rates, the result that recertification can serve as a new "initial" time would be altered. Much can be learned about the tradeoffs faced by program administrators by considering a model in which the failure distributions are exponential even if that assumption does not hold precisely in actual data. The CD is an explicit model that formally captures several factors that determine optimal CDs, and the assumption that the failure distributions are exponentials is adopted to provide a simple structure to consider those factors.

A Single Recertification. To introduce the apparatus of the CD model, this section continues with a simplified CD model that, like the B-P model of the last section, examines the optimal timing of a single recertification. The single recertification will occur at time T to be chosen by the program agency.[29]

The per-certification cost is still given by *c*, for which the present value as of time 0, with ρ as the instantaneous discount rate, is D(T):

$$(4.10)\; D(T) = e^{-\rho T}\, c$$

Immediately from (4.10) it is clear that discounting introduces a consideration that was not present in the B-P model. A delay in recertification yields a marginal gain in terms of lowering D(T) through postponement of the recertification cost *c*. This interpretation of the source of a gain from delay will be central below when interpreting the first-order condition of the problem.

Let U(T) measure the expected discounted unwarranted benefits that cumulate up to recertification time T. $P_{EI}(t)$ registers whether the instantaneous program benefit *m* is unwarranted at any given moment, which means that F(t) in (3.5) of the B-P model needs replacing, resulting in:

$$(4.11)\; U(T) = m\int_0^T e^{-\rho t}\, P_{EI}(t)\, dt = m\int_0^T e^{-\rho t}\left[\frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]dt$$

Upon evaluation of (4.11), U(T) is given by

$$(4.12)\quad U(T) = \left(\frac{\lambda}{\lambda+\mu}\right)m\left[\left(\frac{1}{\rho}\right)\left(1-e^{-\rho T}\right) - \left(\frac{1}{\rho+\lambda+\mu}\right)\left(1-e^{-(\rho+\lambda+\mu)T}\right)\right]$$

The present value of an infinite stream of *m* is *m*/ρ. Importantly, the value of U(T) in the limit takes on a smaller value that equals:

$$(4.13)\; U\hat{} = \lim_{T\to\infty} U(T) = \left(\frac{\lambda}{\rho+\lambda+\mu}\right)\left(\frac{m}{\rho}\right)$$

Thus, if no inspection were ever to occur—i.e., if T were infinitely delayed—the expected unwarranted benefits would be less than *m*/ρ because (in expectation) only a *portion* of the flow of payments of *m* are paid to a client during times of ineligibility. The value that U(T) approaches will be closer to (*m*/ρ) as λ is larger and μ is smaller relative to ρ, inasmuch as those income volatility parameters capture the rates at which clients transition into and out of ineligibility.

[29] In this section's infinite-horizon model, T represents certification time rather than "terminal" time as in the finite-horizon model of the previous section.

Just as the B-P model was used to select an optimal recertification time x* (within a finite period), this single-recertification CD model considers what optimal time T* (within an infinite-horizon context) would minimize expected discounted total cost, given by:

$$(4.14)\ E[DC(T)] = U(T) + D(T) + e^{-\rho T}\left[P_{EI}(T)\,0 + P_{EE}(T)U_{\infty}\right]$$

In (4.14), a flow of unwarranted benefits is accruing, in expectation, until time T. The present value is given by U(T). At T, a recertification cost is paid (with certainty) for which the present value is D(T). At recertification the client is found to be ineligible with probability $P_{EI}(T)$, in which case payments are terminated (making future payments equal 0) and the problem is over. Or, with probability $P_{EE}(T)$, the client is found to be eligible and the payment of program benefits forever begins. That is, if the client passes the problem's *single* recertification, the flow of *m* continues uninterrupted from T onwards without limit. The expected portion of the *unwarranted* benefits portion of that flow, as of and discounted to T, is given by U^. Discounting the two probability-weighted outcomes of recertification at T (of zero future payments and of U^) back to initial time 0, to express them in p*resent* value terms, completes (4.14).

Minimization of (4.14) results in the first-order condition:

$$(4.15)\ P_{EI}(T^*)m - \rho c + P_{EE}'(T^*)U_{\infty} - \rho P_{EE}(T^*)U_{\infty} = 0$$

The problem's second-order condition for minimization is:

$$(4.16)\ P_{EI}'(T^*)m + P_{EE}''(T^*)U_{\infty} - \rho P_{EE}'(T^*)U_{\infty} > 0$$

As in the B-P model, the first-order condition (4.15) can be re-expressed in terms of marginal gains and marginal losses from delayed recertification. At the same time, the first-order condition can be written using a single state probability path and its derivative. Noting that that $P_{EI}(t) = [1 - P_{EE}(t)]$ and using the relationship $P_{EI}'(t) = - P_{EE}'(t)$, (4.15) becomes:

$$(4.17)\ P_{EI}'(T^*)U\hat{} + \rho c + \rho[1 - P_{EI}(T^*)]U\hat{} = P_{EI}(T^*)m$$

The first term shows a gain from delayed recertification: a slight delay in T* increases the probability that ineligibility will be detected at the recertification, thereby saving U^. The second term is new. It captures an additional gain from delaying payment of the per-recertification cost *c*: that cost will be paid sooner or later, but if it is later then in effect the implicit "interest earnings" from delayed payment equal ρ*c*. That is, if *c* were invested "in the bank" earning instantaneous rate ρ until recertification, then proceeds from waiting until the next instant for recertification and earning ρ meanwhile on the invested "capital" of *c* is ρ*c*. In (4.17) there is another gain associated with interest earnings. As of T, there is a probability $[1 – P_{EI}(T)]$ that the client is eligible and that at recertification the client will be due *m* forever, which is tantamount to a lump-sum cost (in present value as of T) of U^. A slight delay in recertification means that that lump-sum value is not paid yet. The lump-sum value can be invested in the bank to accrue interest, resulting in the third term. The term on the right-hand side is the instantaneous cost of delayed recertification, familiar from the B-P model. That cost results from *not* terminating the stream of *m* for a client who is already ineligible (with probability $P_{EI}(T)$). The optimal T* in (4.17) balances these four terms.

Making use of (4.13) to express U^ in (4.17) as a function of *m* (and the parameters λ, μ, and ρ) and then dividing (4.17) throughout by *m* reveals an important feature of the problem. At the optimum, the only dependence T* has on recertification costs and on program benefits is through the term (ρc/m). The numerator of (ρc/m), as noted, is the monetary gain from delaying recertification an instant. The denominator is simply the instantaneous flow of program benefits. This flow constitutes a "potential" cost of delay: *m* is the flow value of unwarranted benefits, but only if the client is ineligible. The *probability* that *m* is unwarranted does matter for determining T*, but the probability aspects appears elsewhere in the (4.17) through the income volatility parameters λ and μ that are contained in $P_{EI}(t)$ and $P'_{EI}(t)$. Here it stressed that *c* and *m* each influence T* through the simple ratio (ρc/m), i.e., through their *relative* monetary values.

Structure of the CD Model. The simple CD model for a single recertification resembles the tree-cutting model of Fisher. Both models assume an action is taken once—recertification or tree harvesting—and the problem is over. The complete CD model developed next resembles Faustmann's solution to the tree-cutting problem. Samuelson (1976) revived and examined Faustmann's solution, which took into account that once a tree is harvested a new sapling can be planted on the land in its place, resulting in yet another possible harvesting to be followed by yet another. The forestry economics literature reached a milestone with the recognition that the optimal harvest time of the first tree-planting cycle should take into account the effect of lengthening or shortening the tree's lifetime on future trees' optimal lifetimes in future cycles. The complete CD model, developed next, incorporates two alternative consequences of any recertification: if the client is ineligible, payments of *m* are terminated; if the client is eligible, payments of *m* are continued until next recertification, and so on across multiple stochastic cycles.

Suppose the program agency determined at time 0 the optimal length of the *first* certification duration to be T_1*. Consider next the program agency's problem as of time T_1* of determining the *second* certification duration, T_2* (for a client who is eligible at that time). Due to the use of an infinite-horizon model and the Markovian assumption, the problem as of time T_1* for the *second* certification duration resembles precisely, in its stochastic specification, the problem as of time 0 for how long to specify the *first* certification duration. Thus, the problem as of time T_1* will yield the same solution, i.e., T_2*= T_1*. Moreover, each certification duration will be the same optimal length. The problem can be characterized has having recertifications occuring at T*, 2T*, 3T*, etc. Although there are multiple cycles, with multiple *dates* of recertification, the problem condenses to the selection a single optimal certification duration T*. The program agency will want to take all future cycles into account when identifying that common value of T* for all cycles, including the first cycle.

Let the expected discounted cost the (first) Single cycle from 0 to T be given by E[S(T)]:

$$(4.18)\ E[S(T)] = U(T) + D(T)$$

Due to the Markovian assumption, the expected discounted cost of any given cycle—discounted to the start of that cycle—depends only on the *length* of the cycle, measured by T, and is independent of calendar time. Thus, the *within-cycle* discounted cost of the second cycle from T to 2T is E[S(T)], as is the within-cycle discounted cost from 2T to 3T.

Because E[S(T)] appropriately discounts within-cycle costs only to the *start* of the cycle, the *present* value of the second cycle's cost involves discounting that value back to time zero, i.e., the present value of the second cycle (conditioned on the client being eligible at T) would be given by

$$(4.19)\ PV\{Cycle\ \#2 \,|\, eligibilty\ at\ T\} = e^{-\rho T} E[S(T)]$$

More generally, the conditional present value for the second and future cycles, n = 2, 3, 4, …, is given by:

$$(4.20)\ PV\{Cycle\ \#n \,|\, eligibilty\ at\ (n-1)T\} = e^{-\rho nT} E[S(T)]$$

Before specifying the full multiple-cycle model, additional features of the problem bears discussion. In (4.19) the present value of the second cycle equal the (discounted) payment of unwarranted benefits within the second cycle and conducting a recertification at the end of the second cycle, conditioned on the client being eligible at recertification at time T. As of time zero, that outcome is uncertain, i.e., it is not unknown whether or not the client will be found eligible at T. Only with a probability $P_{EE}(T)$ will the client be found to be eligible, resulting in E[S(T)] for the second cycle. With probability $P_{EI}(T)$ the client is ineligible, benefits are terminated and the problem is over without a second cycle. Thus, the *expected* present value of the second cycle, would be given by:

$$(4.20)\ EPV\{Cycle\ \#2\} = e^{-\rho T}\left[P_{EE}(T)E[S(T)] + P_{EI}(T)(0)\right]$$

The *third* cycle commences if the client passes the second recertification, and that possible only if the client has already passed the first recertification at the start of the *second* cycle. Recognizing that the probability of passing the first recertification is $P_{EE}(T)$ means

$$(4.21)\ EPV\{Cycle\ \#3\} = P_{EE}(T)e^{-\rho T}\left[P_{EE}(T)E[S(T)] + P_{EI}(T)(0)\right]$$

The *fourth* cycle is reached on condition only with the probability that the third cycle was begun, an event that occurs with probability $P_{EE}(T)^2$. Most generally,

$$(4.22)\ EPV\{Cycle\ \#n\} = \left[P_{EE}(T)\right]^{n-1} e^{-\rho nT}\left[P_{EE}(T)E[S(T)] + P_{EI}(T)(0)\right]$$

where the term $[P_{EE}(T)]^{n-1}$ reflects the number of recertification that must be passed to reach the start of any given cycle.

Let M(T) be the expected discounted cost across Multiple cycles of recertification. M(T) is a collection of costs E[S(T)] associated with each Single cycle. Specifically, M(T) is the collection of the expected present value of each successive cycle:

$$\begin{aligned}(4.23)\ M(T) &= E[S(T)] + e^{-\rho T}\left[P_{EE}(T)E[S(T)] + P_{EI}(T)(0)\right] + P_{EE}(T)e^{-\rho 2T}\left[P_{EE}(T)E[S(T)] + P_{EI}(T)(0)\right] \\ &\quad + [P_{EE}(T)]^2 e^{-\rho 3T}\left[P_{EE}(T)E[S(T)] + P_{EI}(T)(0)\right] + \cdots \\ &= E[S(T)] + [e^{-\rho T}P_{EE}(T)]E[S(T)] + [e^{-\rho T}P_{EE}(T)]^2 E[S(T)] + [e^{-\rho T}P_{EE}(T)]^3 E[S(T)] + \cdots \\ &= \frac{E[S(T)]}{1 - e^{-\rho T}P_{EE}(T)}\end{aligned}$$

The final line in (4.23) follows from viewing M(T) as the infinite sum of terms that involve a geometric sequence.[30]

For some purposes, it will be helpful to express E[S(T)] by its two components, U(T) and D(T), in order to show M(T) as the sum of two cost curves, as shown in (4.24) and Figure 1:

$$(4.24)\ M(T) = \frac{U(T)}{1 - e^{-\rho T}P_{EE}(T)} + \frac{D(T)}{1 - e^{-\rho T}P_{EE}(T)}$$

Effect of Income Volatility on the Cost Curve M(T;λ) Before solving the model, it is helpful to consider the effects of income volatility on the structure of the cost curve M(T) evaluated at any given T, including non-optimal certification periods. Although there are two income volatility parameters, λ and μ, a way to

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[30] The structure of a geometric sequence emerges in contexts besides forestry economics and its successive re-planting and harvesting of trees. Carlton and Perloff (1994), who consider a durability problem in which the (non-stochastic) lifetime of a light bulb N can be increased, at some present cost C(N), in order to achieve savings across an infinite sequence of future light bulb replacements. Greenfield and Persselin (2002) examine the replacement of military aircraft across multiple "generations" of aircraft.

examine the effect of income volatility is simply by increasing λ holding other factors (including μ) constant. It is helpful to re-express (4.23) and (4.24) as a functions of both T and λ, as in M(T; λ).

An increase in λ has two effects: an *unwarranted-benefits effect* that tends to increase M(T; λ) and a *recertification-effect* that tends to decrease M(T; λ). It might intuitive that an increase in λ would raise M(T; λ). After all, unwarranted benefits result from income volatility: an increase in λ rotates upwards the state probability path $P_{EI}(T; \lambda)$ (the path's intercept remains fixed at 0), thus increasing expected discounted unwarranted benefits U(T; λ) at any given T, and increasing E[S(T; λ)] and M(T;λ) in turn. However, there is a second effect to consider. The same increase in $P_{EI}(T; \lambda)$ that increases unwarranted benefits also raises the chance of detecting an ineligible client *at each recertification*, an outcome that would result in terminating the client's participation and saving on (expected) future costs (of both unwarranted benefits and recertification costs). Inasmuch as this effect of λ on M(T; λ) operates through affecting the expected number of recertifications, it is called here a "recertification effect." Formally, the affect shows up in the denominators of (4.23) and (4.24) which contain the term $P_{EE}(T; \lambda)$ in the first place because M(T; λ) was derived by considering an infinite sequence of *possible* cycles following *possible* recertifications. The increase in $P_{EI}(T; \lambda)$ that results from an increase in λ is tantamount to a decrease in $P_{EE}(T; \lambda)$.

The relative sizes of the unwarranted benefits effect and the recertification effect can be established. Taking the natural logarithm of (4.23) and differentiating results in:

$$(4.25)\ \frac{\partial \ln M(T;\lambda)}{\partial \lambda} = \frac{1}{E[S(T)]}\int_0^T e^{-\rho T} m \frac{\partial P_{EI}(T;\lambda)}{\partial \lambda} dt - \left(\frac{1}{1-e^{-\rho T}(1-P_{EI}(T))}\right)\left(e^{-\rho T}\frac{\partial P_{EI}(T;\lambda)}{\partial \lambda}\right)$$

$$= \left(\frac{\partial P_{EI}(T;\lambda)}{\partial \lambda}\right)\left(\frac{M(T;\lambda)}{1-e^{-\rho T}P_{EE}(T)}\right)\left[\frac{m}{\rho}\left(1-e^{-\rho T}\right) - Me^{-\rho T}\right]$$

It is straightforward to show that $P_{EI}(T)$ increases with λ, as expected, making the sign of the effect of λ on M(T; λ) equal to the sign of the terms inside the brackets. As T approaches zero, the terms in the brackets approach –M, making (4.25) negative. As T increases without limit, the terms in the brackets approach (m/ρ), making (4.25) positive. If the terms in the brackets are set equal to zero, a particular certification length T_0 can be derived at which the derivative is zero, i.e., at T_0 the value of $M(T_0; \lambda)$ is unaffected by income volatility.

An illustration of one possible outcome is shown in Figure 2. The M(T; λ) curves associated with some given level of income volatility is M1(T), and M2(T) results from an increase in income volatility. The values of the M2(T) are below the values of M1(T) for $T < T_0$, but they exceed the M1(T) values for the time range $T > T_0$. This result means that the recertification effect dominates the unwarranted-benefits effect at low values of T (i.e., $T < T_0$). An increase in income volatility would shift *down* the cost curve in this time range. It is fully plausible that the cost savings from potentially cancelling recertifications is especially large when recertifications are frequent (a short T). In contrast, at relatively long CDs, (i.e., for the range $T > T_0$), the unwarranted-benefits effect dominates the recertification effect. The accumulation of unwarranted benefits is so strongly affected by an increase in λ—the effect is strong precisely because the certification period is so long—that it outweighs the savings from the recertification effect. In this time range, the cost curve would shift *up* when income volatility increases. However, it may be best not to use the terms "shift down" or "shift up" because of their common associated with movements down or up of entire cost curves. Instead, the most complete description may be that an increase in income volatility *twists* the structure of the cost curve.[31]

Solution of the CD Model. The solution to (4.23) follows from minimization of M(T) with respect to T to obtain optimal certification duration T* for all cycles, with each cycle taking into account the probabilistic possibilities of future cycles.

An alternative way by which to derive the solution makes use of Bellman's concept of optimality in dynamic programming. Let V(T*) be the expected discounted costs across all cycles if the optimal duration T* is in use, i.e., V(T*) is given by M(T*) where M(T) in (4.11) is evaluated at its solution T*. That is, V(T*) is the (minimized) *optimal* value of the expected discounted cost. During the first cycle, the expected discounted cost evaluated at the optimal T* is given by E[S(T*)]. At the end of the first cycle, at time T*, there is a chance of $P_{EI}(T^*)$ that the client is ineligible and payments of m cease. There is a probability $P_{EE}(T^*)$ that the client is eligible and benefits continue. As noted before, the situation at time T* is identical in its stochastic characteristics to the initial situation at time 0 except that it is T* later. Thus, the expected discounted cost of the potentially infinite number of all future cycles (on the premise that the optimal duration T* is in use forevermore) from the *second* cycle onwards is again V(T*), albeit evaluated at time T*. To re-value that cost as of the present requires that it be discounted to time 0. Therefore, the initial V(T*) can be expressed as the sum of E[S(T*)] and the discounted value of V(T*) itself, where the discounting occurs over the period of one single cycle:

[31] An alternative illustration, for a different scenario and different model parameters, might show T_0 along the portion of the M1(T) that is downward-sloping rather than (as in the figure) upward sloping.

$$(4.26) \quad V(T^*) = E[S(T^*)] + e^{-\rho T^*}\left[P_{EI}(T^*)(0) + P_{EE}(T^*)V(T^*)\right]$$

The solution to the problem can be found by differentiation of (4.14) with respect to T, yielding a first-order condition that holds only at the optimal T*. That first-order condition is given by

$$(4.27) V'(T^*) = E[S'(T^*)] + e^{-\rho T^*}\left[P_{EE}(T^*)V'(T^*) + P_{EE}{}'(T^*)V(T^*)\right] - \rho e^{-\rho T^*}\left[P_{EE}(T^*)V(T^*)\right]$$

Rearranging (4.27) to express V'(T*) in terms of the other expressions, and then setting V'(T*) equal to zero to find the optimal T* gives:

$$(4.28) \quad 0 = V'(T^*) = \frac{E[S'(T^*)] + e^{-\rho T^*}\left[P_{EE}{}'(T^*) - \rho P_{EE}(T^*)\right]V(T^*)}{1 - e^{-\rho T^*}P_{EE}(T^*)}$$

Direct minimization of M(T) in (4.23) shows that the solution 0 = M'(T*) is the same one derived by dynamic programming in (4.28).

The Appendix derives certain operating characteristics of the CD model, which describe various outcomes of the system that are of interest. The next section that simulations optimal CDs makes reference to operating characteristics.

Three first-order conditions have been presented: (3.9) for the B-P model, (4.17) for the single-recertification case of the CD model, and (4.28) for the (multiple-cycle) CD model. These are useful for characterizing the properties of optimal solutions and the factors that influence them. However, a first-order condition is not always the most straightforward way to calculate the optimal solution. The optimal CDs in the simulation were obtained by constructing M(T) directly, based on (4.23) and the functions and parameters in it, and then letting T increase in one-month increments. (An upper limit of 99 months was examined.) The optimal CD, at T*, is identified as the last month such that the change M(T*) – M(T*-1) is negative An additional month past T* results in an increase in M(T), so M(T) is minimized at T*.[32]

5. Application: WIC Children's Certification Duration

This section uses the CD model to simulate optimal T* from three perspectives—Government Budget Perspective, Taxpayer Perspective, and Social Perspective. The certification period for children participating in WIC is chosen as the case study. Although WIC participants must be determined to be at

[32] In contrast, the model examined in Greenfield and Persselin (2006, p. 11) was reportedly solved numerically using *Mathematica*, presumably making use of the first-order condition.

nutritional risk, the stochastic process by which individuals enter or exit the state of nutritional risk is not considered here. Instead, the focus is entirely on income volatility.

There are five group of WIC participants: pregnant women, breastfeeding women, non-breastfeeding postpartum women, infants (up to one year old), and children (1 through 4 years of age). The CD issue can be considered more salient for WIC children than for other WIC clients. First, children have the longest period for which they could be in their eligibility category—up to four years. Children are currently recertified in WIC every six months, which constitutes potentially several recertifications over a full four-year participation period. The other participant categories have much less potential for duration of participation. So it is of particular interest to ascertain whether the CD model suggests more frequent or less frequent recertifications may be optimal for WIC children. A second reason for using WIC children for a case study that children are half of the program's participants.[33]

Conducting any simulation exercise involves specifying an economic model and assigning values for key parameters. Selecting values for the CD model's parameters for WIC may may have greater uncertainty than for other programs such as the FSP. For example, while there is some information about certification activities for local WIC agencies (e.g., Kuchak and Pindus, 1995) certification costs capturing the time involved, from an agency and a client's perspective, have not been studied as much for WIC as for FSP, e.g., by Hamilton, Burstein, Hart, Bartlett, Sullivan, and Teixeira (1989) and Ponza, Ohls, Moreno, Zambrowski, and Cohen (1999).[34] In addition, findings on income volatility of families with WIC children are sparse and information on the dollar cost of the WIC food package for children is incomplete. This section explains the assumptions that have been made to develop a set of values for model parameters. The resulting simulations for optional CDs are tentative, subject to further refinement by model improvements or by the use of alternative parameter values that may be more appropriate than those proposed here.

Timeframes for Income Eligibility in WIC. An expert panel convened by the National Academy of Sciences (NAS) examined methods of estimating eligibility and participation in WIC. The report of the NAS panel quoted instructions from the USDA to state and local WIC agencies:

[33] In fiscal 2005, the average annual participation figures were 1,966,249 women; 2,047,118 infants; 4,009,248 children. (www.fns.usda.gov/pd/wic_monthly.htm)

[34] Because the FSP involves more participants, greater total funding, and (often) greater monthly benefits than WIC, the disparity between the scope of research effort for the two programs can be a reasonable response to limited research budgets.

"In determining the income eligibility of an applicant, the State agency may instruct local agencies to consider the income of the family during the past 12 months and the family's current rate of income to determine which more accurately reflects the family's status. . . [I]n the case of families of self-employed persons, including farmers or seasonally employed persons whose income fluctuates, annual income may be the more appropriate indicator of the need for WIC benefits." (Ver Ploeg and Betson, 2003, p. 53)

For the "current" rate of income, monthly or even weekly income may be considered. The questions then become: What timeframe seems to be most consistent with the program's intent? What timeframe seems to be used the most in practice?

In answer to the first question, the NAS panel designated the "previous month" as the appropriate time period to "represent the intent of the WIC regulations." (Ver Ploeg and Betson, 2003, p. 54).[35] In answer to the second, a study by Gordon, Lewis and Radbill (1997) on income volatility n WIC families states that "WIC program staff members in general use monthly income to evaluate eligibility" and that "most of the children who appear ineligible on the basis of annual income may well have been eligible on the basis of monthly income at the time of certification." (pp. xiii, 67). A similar assessment was made in the study by Bitler, Currie, and Scholz (2003), who stated, "incomes fluctuate frequently over the year and people may join the program when their incomes are temporarily low . . . Although WIC offices may use annual income in some circumstances, we believe that monthly income more closely approximates the concept of income that is generally used in practice." (pp. 1156, 1158)

An implication of adopting a timeframe of one month as "appropriate" (in general) for determining income-eligibility is that post-application increases in *monthly* income may be interpreted as moving the WIC family out of income-eligibility. This implication was recognized explicitly by the NAS panel, who noted "Accounting for monthly income, certification periods, and adjunctive eligibility has a large impact on the estimated number of eligible persons. This might raise concerns that the 6-, 9-, and 12-month certification periods allow some people who gain eligibility for WIC due to one or two months of low income or means tested program participation to continue to be certified for WIC for month in which they are not income or adjunctively eligible." (Ver Ploeg and Betson, 2003, p. 65)

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[35] It is clear that the intention is not to guarantee program benefits for the entire certification period regardless of any changes to the client's circumstances. Federal regulations (7 CFR 246.7(h)) state that "The State agency shall ensure that local agencies disqualify an individual during a certification period if, on the basis of a reassessment of program eligibility status, the individual is determined ineligible." In the absence of information on annual income, the local agency would act on the basis of information on "current" income.

The underlying premise of the CD model is that a client eligible at certification could stochastically exit from income-eligibility in a later time period. But the model was silent on the question of "timeframe." The information above suggests that adopting a timeframe of a month is a reasonable one for modeling. Therefore, whether WIC benefits are warranted or unwarranted will be determined with reference to income during a given month.

Monthly Income Volatility in WIC households. A motivation for much of the previous work on income volatility in WIC has been to inform and potentially improve the measurement of WIC participation rates, i.e., the proportion of eligible persons (by category) who choose to apply for and participate in WIC. There is a longstanding awareness that income volatility can affect the usefulness of Current Population Survey (CPS) data for measuring participation rates because annual income, as reported in the CPS, may not fully capture within-year eligibility for WIC households.

Findings from several previous studies suggest the possibility of the payment of some amount of unwarranted benefits to WIC children. Table 3 shows some of the results of the study by Gordon et al. The report found (using SIPP for 1990-92) that for about 17 percent of families with children (ages 1-4), monthly income is sufficiently low in some months that they are eligible for part of the year, but their income in other months is so high that they are ineligible based on annual income. In addition, about 15.7 percent of families with children are eligible in some months and, even though they are not eligible in all months, their cumulative household income for the year is sufficiently low that they are still annually eligible. These two groups combined represent one-third of families with children (32.7 percent) who are income eligible at least part of the year but are ineligible in other months. (About one-fourth (26.5 percent) of families with children are income-eligible for WIC every month of the year; unwarranted benefits would be zero for these children if they were WIC participants.) These estimates are for all families with children rather than for families with WIC children. If the income dynamics are similar for the two types of households, then perhaps up to one-third of WIC children can be income eligible at the time they certify and exit income eligibility before the calendar year is over.

A second source on income volatility in WIC households is U.S. Department of Agriculture (2001). According to the National Survey of WIC Participants, a special survey conducted in 1998, about 75.1 percent of all WIC households were employed at the time of a WIC certification, and 74.2 percent were employed approximately four months later. The similarity of the two employment figures at the two times is due in part to the cross-sectional nature of the statistic. The same report indicates that, if individual households are followed separately, 9.8 percent of all WIC households entered employment in the 4-month period, while 10.7 percent exited employment: the net change of -0.9 percent accounts for the 0.9 decrease in the cross-sectional employment figures. Of the 9.8 percent of WIC households that entered employment, at least some are likely to have included WIC children and gained sufficient income (by the very act of entering employment) that the household was not income-eligible at some point within the four-month period.

The NAS panel used 1997 SIPP data to allocate eligible children (results were reported separately for infants) across three categories. The unit of analysis was the "case-month" rather than the child or the household. There were 14.5 percent of children's case-months that were in the category of "months in which an infant or child was certified as eligible but not eligible that month based on that month's income, nor annual income or adjunctive eligibility." (p. 66) In addition, there are 11.2 percent of children's case-months belonging to children "whose numbers of months of eligibility are less than the number of months they would be certified as eligible but who have annual incomes below 185 percent of poverty or who report participation in means tested programs and are adjunctively eligible." (pp. 65-66).

Bitler et al. examined how the number of children *measured* as "eligibile" for WIC varied according to several factors (separately and in combinations), including the use of monthly or annual income, and the roles of adjunctive eligibility and the certification period. The study used SIPP to develop separate figures for 1997 and 1998. Only the 1997 figures are reported here. The average monthly number of children eligible for WIC on the basis of annual income was 6,744,049. This figure rises to 7,119,389 based on monthly income, under which "a person is only eligible in the single month that they meet eligibility guidelines." (pp. 1161-1163). Taking adjunctive eligibility into account brings the total to 7,630,879. Finally, a key contribution of the study from the point of view of this paper, was the separate effect due to certification period, which increases the number counted as "eligible" (in the language of Bitler et al.) because those will be counted due to being certified even when not income-eligible or adjunctively eligible.[36] The certification effect brought the measured number of "eligible" children to 9,383,579, an increase of 1.75 million children, or 23 percent greater than the number eligible based on monthly income and adjunctive eligibility alone.

[36] Bitler et al. refers to children as "eligible" so long as the children are certified, even though they are not income-eligible nor adjunctively eligible. This paper refers to such children as "certified" to receive program benefits but "ineligible" to receive them in the month(s) in question.

Although the three studies did not each examine the same year—1990-92 in Gordon et al., 1997 in NAS and Bitler et al.—and did not each adopt the same "unit of analysis"—family or case-month or child—the figures from the various studies are in some proximity. Gordon et al. found a figure of one-third (32.7 percent) when counting families with children who are eligible for just part of the year. The NAS study found a figure of one-fourth (25.7 percent) when counting the percentage of children's case-months that are eligible for just part of the year. Bitler et al. found a figure of one-fourth (23 percent) when counting the additional number of children who are certified for part of the year even though (in the language of the CD model) they are ineligible for that part.

While these findings strongly indicate the presence of unwarranted benefits, for two reasons the findings are suggestive, rather than conclusive, about the *scale* of unwarranted benefits. That it, it may be that the share of unwarranted benefits out of all WIC benefits (to children) is not well-measured by the figures one-fourth or one-third. First, the sample each report studied was not restricted to WIC children. This paper does not overcome that limitation in its simulations. Second, the *timing* of months of WIC participation relative to the months of eligibility and ineligibility may be critical. Months of eligibility and ineligibility are not distributed completely randomly within the year, but instead are clustered. Tallying counts of out-of-eligibility case-months or out-of-eligibility families or children is not fully equivalent to counting months in which unwarranted benefits are received prior to termination at certification. When an ineligible child undergoes WIC recertification, the program terminates benefits, cutting short the series of months in which unwarranted benefits are paid—indeed, that is the goal of the process.[37] Out-of-eligibility case months that follow termination of program benefits do not add to unwarranted benefits. In the end, it could be that a "large" share of WIC children receive unwarranted benefits, but the share of months that they are participating and ineligible, relative to total months of participation, may be "small." The figures of one-fourth or one-third may be better interrupted as upper bounds on the share of program benefits that are unwarranted. This paper's analysis of optimal CDs is designed specifically to examine the interplay of the timings of ineligibility and certification using the state probability path $P_{EI}(t)$ and the certification duration T.

Ideally, there would be information on the state probabilities, month by month, by which a WIC child who is certified eligible at some time stochastically becomes ineligible at later times. The findings summarized above indicate the important role that income volatility plays in affecting measures of the numbers "eligible" for WIC. But these studies have not considered income volatility using the stochastic

[37] In the FSP, a second goal of recertification is to adjust benefits to a new warranted level for a client who is eligible but has a new level of income. In WIC, program benefits for eligibles do not depend on income.

time profiles needed by the CD model. That is, there is no research to draw upon that directly measures $P_{EI}(t)$ for households with WIC children. An alternative set of state probabilities paths is used here as a proxy. The simulation of the CD model will be presumed that an estimate of $P_{EI}(t)$ for households with *school-aged* children matches the path for households with *WIC* children (ages 1 through 4).

A defense of this assumption begins with a possible concern, known from earlier research, that income is volatile for women surrounding the birth event. At first, then, it may seem to be a large stretch to assume that the state probability paths for households with school-aged children match the paths for households with WIC children. However, research findings suggest that many mothers who had been employed before pregnancy return to work within the first year after giving birth. Based on data for young women in the National Longitudinal Survey of Youth, Klerman and Leibowitz (1990) find that over one-third of mothers returned were at work within 3 months following birth, and three-quarters within two years. The simulations of the paper presume that the volatility surrounding the birth event is sufficiently resolved by the end of the first year that the birth event does not substantially effect the probability paths for households with WIC children.

It is recognized that income volatility for households could depend on the ages of the child(ren). After all, households with older children are likely to have adults who are systematically older—it is not just the children who are aging—and may as a result have different labor market experiences and outcomes. Furthermore, the different ages of the child(ren) may affect labor market behavior even for adults of the same age. In addition, an ideal data set would have the probability paths not for households with WIC-aged children per se but for households with WIC-participating children.[38] Nevertheless, a useful starting point for research in this area is this paper's use of a month-by-month state probability paths even though the households' children are older rather than young and WIC participants.

Although 185 percent of poverty is the income-eligibility threshold for WIC, some WIC participants reside in households with higher income and are still eligible for WIC due to the adjunctively eligible rule: persons who participate in Medicaid are adjunctively eligible for WIC, and some States permit households with income above 185 percent of poverty to enroll in Medicaid.[39] The application of the CD model in this section ignores variation in State Medicaid policies, and treats 185 percent as a uniform threshold for simulation purposes.

[38] There may be a systematic difference in income volatility between households with WIC program participants and non-participants. Such a phenomenon was found for FSP participants and non-participants in Farrell et al. (2006).
[39] Adjunctive eligibility is also possible for WIC through participation TANF or food stamps, but those two programs have income eligibility limits below the 185 percent threshold for WIC.

State Probability Paths of Households With Older Children. A longitudinal approach is needed by which the income trajectory of individual households are followed over time in order to identify which consecutive months a household is eligible and which months a household is ineligible. The simulations draw on results by Newman (2006), who identified a set of households that are income-eligible at a particular moment and followed that set over time to examine, for the set as a whole, what percentage were ineligible in each successive month. That approach yields a stochastic time profile that represents the state probability path $P_{EI}(t)$ needed for the simulation. It so happens that the Newman study contained three such moments of eligibility, each for the month of August in three successive years, 1996 through 1998; the month of August was chosen because Newman was examining school meals issues. The first three columns of Table 4 show Newman's results, by year; the data make use of the "seam-adjusted" share figures reported in Newman.

In month 0 (August of each respective year), every household was eligible, making 0 percent ineligible. Then, as the months (of the school year) passed, the percentage of households that are ineligible, at a given month, increases at a decreasing rate. The table also shows, in the fourth column, the monthly averages across the three years' state probability figures to obtain an "average" state probability path. The average path approaches a value that is taken to be a "steady-state" value. The transitional nature of the average probability path is seen more readily in Figure 3 which depicts graphically the table's average path figures.[40] In order to separate and estimate implicit exponential exit and re-entry rates for eligibility, the steady-state probability $\mu/(\lambda+\mu)$ toward which the state probability path approaches was set (by visual inspection) at 0.25. The rate of approach towards the steady-state probability was used to estimate the rate of approach of the state probability path using OLS. From the (5.8) it follows that

$$(5.1)\ \ln\left(1-\frac{P_{EI}(t)}{0.25}\right)=-\left(\lambda+\mu\right)t$$

where the average figures of column 4 are used for the data on $P_{EI}(t)$ in (5.1). The coefficient on time was estimated to be -0.309 (with the restriction of zero-intercept imposed), with an R-squared value of 0.85 for the equation. Based on the estimated coefficient, the estimated (monthly) value of λ was 7.725 percent, while μ was estimated at 23.175 percent. It is no accident that the estimated value of μ is precisely 3 times the estimated value of λ: that relationship implies and is implied by the stipulation that the steady-state probability $\mu/(\lambda+\mu) = 0.25$, and serves as the identification restriction by which the two structural parameters are obtained from the estimated coefficient on time.

The fifth column of the table and its accompanying figure each show a "fitted" state probability path, based on the estimates of the two structural parameters. Other estimates of λ and μ are also used for a sensitivity analysis.

Dollar Value of WIC Children's Benefits. This paper does not account for the dollar value of WIC's nutrition counseling and health referrals when conducting the simulations: the value of the WIC food package alone is considered.

The CD for WIC children potentially affects about half of the WIC programs food costs, which in fiscal 2005 were $3.6 billion.[41] This result does not follow from the observation alone that children comprise half of WIC's participants: the children's food package could cost relatively more or less than other WIC food package. (Each category of WIC participant receives a specific food package.) The children's package contains such items as cereal, peanut butter, juice, cheese, milk, and eggs; participants have some choice regarding combinations of certain items. Based on data from Information Resources, Inc. (IRI) on supermarket transactions for about 43,000 households in 1997-1999, an estimate (explained below) of the WIC food package cost for children is $32.52. The average monthly WIC for cost per participant, in 1997-99 respectively, was $31.68, $31.76, and $32.50. These three figures are not strikingly different from the $32.52 estimate the WIC children's package for those years. Thus, with children constituting half of WIC participants and with the cost of their food package similar to the cost of food packages averaged across participants, children's share of WIC food package cost is about half.

The simulation of the CD model requires a figure for monthly WIC benefits for children. The cost of a WIC for package for a particular participant category is not routinely collected or estimated. The most comprehensive work in this area, by Davis and Leibtag (2005), examined the role of food prices, caseload composition, and cost-containment practices in affecting a State's WIC food package costs for 17 selected States under study. For purposes of defining a common cross-State food package standard by which to compare food package costs, Davis and Leibtag used the *maximum* quantity of food available in each food package. However, while WIC can provide a particular client a prescription of foods up to the maximum set by Federal regulation, WIC tries to tailor the amounts of a packages individual food items to the individual client to match preferences and avoid waste. The average prescriptions are typically below the maximum prescriptions. For example, the federal maximum prescription for milk is 24 quarts,

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[40] One small difference in labels is noted: the first month in the table is month 0, while in the figure it is month 1.
[41] Total food cost and average monthly food cost per participant are at http://www.fns.usda.gov/pd/wisummary.htm

but only 2.1 percent of children received the federal maximum. The average quantity of milk prescribed is 16.8 quarts. To better approximate the pattern of actual prescriptions, this paper took into account, by item, that the (national) average quantity received by a child may be below the federal maximum, resulting in a slight adjustment to the work by Davis and Leibtag. Table 5 shows the estimated value of the children's food package for each of the 17 selected States, based on prices in an IRI "market area" in that State; as Davis and Leibtag note, these prices may not be representative of prices in the entire State. The table shows $32.52 as the simple average across the 17 States' figures.

WIC Recertification Costs. An estimate of agency recertification costs is derived using three factors: a figure for the hourly wage rate for direct staff time, a figure for the direct staff time (in hours) associated with recertification, and a percentage figure for an overhead rate. The cost figure that is derived is for 1998, the middle of the 1997-99 range for which the WIC food package costs were derived and in the 1996-98 range of the Newman income volatility data.

When viewing the public health nutrition workforce in Full Time Equivalent positions, four-fifths (81 percent) are employed in WIC programs (McCall and Keir, 2003). Across the 2,200 local agencies and 10,000 WIC service sites, caseload and staffing vary. Some local agencies have one or two staff while others have over 350 (U.S. Department of Agriculture, 2006, p. 14) Various staff need administrative, nutrition, and medical skills. Tasks involved with certification (as opposed to providing program benefits in the form of nutritional counseling or breastfeeding promotion) include determination of identity, State of residency, and income-eligibility, measuring height and weight, drawing blood and doing analysis, and determining nutritional-risk. At the local level, WIC uses a variety of approaches and different combinations to professionals, paraprofessionals, and WIC's Competent Professional Authorities to conduct the various certification tasks.[42] This report uses Bureau of Labor Statistics (BLS) data to estimate a 1998 figure for total hourly compensation across "health care and social assistance employees" in State and Local Government to proxy what weighted-average combination of employees wages and times may affect certification cost in any particular locality.[43]

[42] A recent study on WIC staffing stated that "Little specific information exists about the actual performance of duties throughout the various classifications within the nutrition workforce, or specifically the WIC workforce." (U.S. Department of Agriculture, 2006b, p.31) The same study notes that "Some clinics attempt to get both anthropometric and blood work from other providers, while some clinics do their own anthropometrics and use blood work obtained elsewhere." (p. 21)

[43] A weighted-average approach would draw upon such information as a 1999-2000 survey of staffing and annual salaries reported by McCall and Keir (2003), in which the median low-salary and the median high-salary for various positions were $18,804-$25,251 for Nutrition Assistant, $20,736-$29,163 for Nutrition Technician, $26,352-$39,000 for Nutritionist, and $29,661-$43,496 for Clinical Nutritionist; five other job titles and salaries were reported too. One difficulty with constructing weighted-average wage rate is that positions with these have non-

The National Compensation Survey of the BLS reports that the cost per hour worked (in terms of wages and salaries) for "health care and social assistance employees" in State and local government average $23.53 in the 2nd quarter of 2006, representing 65.9 percent of total compensation (U.S. Bureau of Labor Statistics).[44] Because BLS recently shifted to the NAICS industrial classification system (NAICS), replacing the SIC system, the time series for NAICS figures begins only in 2004. A NAICS figure for 1998 is not available. However, the BLS figure (for SIC) for 1998 for wages and salaries of "all workers" in State and local government" is $19.19 per hour, representing 70.3 percent of total compensation. Thus, the total compensation in 1998 for "all workers" was $27.30. An adjustment of this figure downward, to better estimate total compensation of "health care and social assistance employees" who staff WIC, results in a estimate of $25.40 of total compensation per hour in 1998.[45] This figure is used as an approximation to a weighted-average wage rate across staff involved with WIC certifications.

Among the documents the Office of Management and Budget (OMB) provides for guidance and reference is a general guide to benefit-cost analysis that states:

> When calculating labor costs, OMB recommends using prevailing wage rates and salaries. To arrive at fully burdened costs when estimating personnel costs for government employees, you must add overhead costs to salary and fringe benefit costs . . . Some examples of indirect costs include rent, utilities, insurance, indirect labor, and other expenses typically charged to the organization as a whole . . . For evaluation purposes, costs (both direct and indirect) should be included if they will change with the introduction of a proposed system. (Capital Planning and IT Investment Committee, 1999, pp. 14-15)

It is sensible to account for indirect costs. A change in certification policy can have long-term and large-scale effects on the caseload (as shown in the previous section) and on the agency's overall staffing and capacity to service that caseload. The CD model was developed to address a program administration

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uniform duties across local WIC agencies. Another is the absence of information on time each contributes to a certification.

44 A check on the correspondence between the hourly wage of an actual WIC staff position and the BLS figure is provided by a job-vacancy notice posted in summer 2006 by Rice County, Minnesota seeking a WIC professional (public health nurse or nutrition professional). The job activities included "all phases of the WIC certification process, including hematological screening." The stated hourly salary range was $19.31 to $26.71, for which the mid-point is $23.01, differing from the BLS figure by about one-half dollar.

45 Quarterly data for 2005 are available for both "health care and social assistance employees" and for "all occupations" (both series in the state and local government sector). Total compensation was computed for each of four quarters of 2005 for the two groups using "cost per hour worked" and "percent of total compensation". The annual average of the quarterly figures is $35.92 for "all occupations" and $33.42 for "health care and social assistance." In 2005, "health care and social assistance" workers received (on average) 93.0 percent of the total

issue that could be considered to be a new "proposed system" (of a longer or shorter CD). Therefore, obtaining a "burdened" or a "loaded" hourly labor cost that includes indirect costs is suitable. The total compensation figure estimated for "health care and social assistance employees" in State and local government was estimated to be $25.40 for 1998. To cover all indirect costs as well, this paper applies an arbitrary overhead rate of 100 percent to the $25.40 total hourly compensation to obtain a loaded hourly labor cost of $50.80.[46]

For the simulation, a figure of 1 ½ hours was used for the staff time involved with a certification. This figure represents a composite of figures from six websites at the county or State level that inform potential WIC applicants how long they need to plan for conducting a certification.[47] It is presumed that the 1 ½ hour figure represents "certification" activity, as opposed to time spent in the provision of non-food WIC benefits in the forms of health referrals, nutrition counseling, and breastfeeding promotion. Separate appointments at the WIC clinic are often made for these other activities.

Although the time for staff is assumed to be 1 ½ hours for certification, the time for a client is assumed to be 2 hours to take into account travel time. As noted before, about three-quarters of WIC households earn a wage or salary. Even those who have other sources of income place a value on their time. An arbitrary figure of $6 per hour is used to estimate the value of time to the person, presumably the mother, who brings the WIC child to the program for certification.[48] An arbitrary figure of $5 per certification is used for out-of-pocket costs for the WIC child's mother.

Social Discount Rate. There is a literature on what the concept of the social discount rate means, or ought to mean, and how to measure it in terms of an observable private market discount rate. A range of 2 percent to 10 percent (in real terms) may encompass most estimates. The Office of Management and

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compensation paid to "all occupations." Applying the 93.0 percent figure to the total compensation for "all workers" in 1998 of $27.30 yields a figure of $25.40 for "health care and social assistance employees" in that year.

[46] Although the Capital Planning committee's guide provides reference to OMB's Circular 76, which contains an "overhead factor" of but 12 percent (to be applied to personnel costs inclusive of fringe benefits), A-76 addresses competitive outsourcing. The paper uses an overhead rate chosen for the purpose of conducting a simulation for changing certification policy.

[47] The six websites were selected based only on convenience, rather than constituting a nationally representative sample. The reported figures were: "about" 30 minutes in Minnesota; 1-2 hours in Michigan; 1-2 hours in Fairfax County, Virginia; "at least" 1 ½ hours in Clinton County, New York; "at least" 1 ½ hours in Utah County, Utah; and 1 ½-2 hours in St. Charles County, Missouri.

[48] Of those households with WIC children, median annual income (all sources) is $15,325 for households with an adult male present, and $8,520 for households without an adult male present in 1998 (exhibit 3-22). The paper takes the $8,520 figure as the annual income of a mother with a WIC child with or without an adult male present. The assumption of $6 per hour is consistent with annual earnings of $6000 and 1000 hours of annual work.

Budget (OMB) annually provides federal agencies with updated discount rates, in support of Circular A-94 (U.S. Office of Management and Budget, 1992) that provides guidance on benefit-cost analysis. As of January 2006, real discount rates for use in cost-effectiveness analysis (as opposed to other decisions) are reported at 2.5 percent for a 3-year horizon up to 3.0 percent for a 30-year horizon. The simulation will use a monthly discount rate of 0.002 for the baseline, and an alternative figure of 0.004.

Excess Burden. OMB Circular A-94 gives a figure of 25 percent for excess burden (ε), although it also provides for the use of other figures. The 25 percent figure exceeds an estimate of 19.5 percent efficiency cost for combined local, state, and federal level taxes relative to a nondistortionary, revenue-neutral tax in a study by Jorgenson and Yun. Their study was reported to be one of the two studies with the "broadest scope" in a General Accountability Office (GAO) review of the compliance and efficiency cost of taxes (U.S. Government Accountability Office, 2005). A study by Ballard (1988), not cited in the GAO report, reports much higher marginal efficiency costs (estimated for financing a universal cash grant program) of 50 to 130 percent. The simulation will use the 25 percent OMB figure to represent excess burden for the baseline, and an alternative figure of 0.195 percent.

Simulation of Optimal CDs for WIC Children. Table 6 provides a summary of each parameter value for the simulation. An optimal CD is re-computed from each of three perspectives, which differ in their use of the parameters in table 1. As discussed in the second section, the Government Budget Perspective focuses on bookkeeping costs, thus treating marginal excess burden as zero. The Taxpayer Perspective incorporates the 25 percent figure for ε shown in the table. The Social Perspective ignores the transfer payment aspect of the program, leaving excess burden as a critical factor. The Social Perspective, unlike the other two, incorporates the recertification cost that the client pays in addition to the agency's recertification cost.

The first two rows of Table 7 show the baseline simulation's figures for recertification costs and "effective" benefits from all three perspectives. The recertification cost from the Government Budget Perspective is ($25.40/hr)(2.0)(1.5 hours) or $76.20, which reflects the 100 percent overhead factor (built into the figure 2.0). The recertification cost from the Taxpayer Perspective is (1.25)($76.20) or $95.25, and the Social Perspective adds an additional $17, reflecting $12 worth of time costs (i.e., ($6/hr)(2 hours)) and $5 out-of-pocket costs, to reach $112.25. The 17-State average WIC children's food package cost of $32.52 is the bookkeeping cost that shows in the Government Budget Perspective. The "effective" benefits figures are $40.65 and $8.13 for the Taxpayer and Social Perspectives, resulting from adjusting m upwards by the factor (1+ ε) or downwards by the factor ε (as done in section 2).

Rows 3 and 4 of Table 7 show the solution to the baseline simulation consists as a pair of values, T* and its associated M(T*), representing the optimal, cost-minimizing CD and the minimized value of total cost that results from using T*. The simulated T* for the Government Budget Perspective is 6 months. Its associated value for M(T*) is $461.65, representing the present value, in expectation, of recertification cost and unwarranted benefits across all potential future recertification cycles. The second column shows T* is also simulated to be 6 months for Taxpayer Perspective, with M(T*) of $577.06. The Social Perspective's recertification cost of $112.25 and effective benefits of $8.13 and result in T* of 12 months and M(T*) of $486.28.

Figure 1 was introduced earlier to illustrate the relationship between M(T) its two component cost curves. The figure contains the numerical curves for an actual case: they represent the curves result for the parameter values of the baseline case for the Government Budget Perspective. A characteristic of M(T) in the figure is that it is asymmetrical: the curve descends more steeply on the left of its minimum than it rate of increase towards the right. Although the various simulations yielded a variety of shapes for M(T), they shared the general property of asymmetry, a property driven largely by the "flattening out" of the curve for expected discounted recertification costs (across all cycles).

The asymmetry of M(T) has an implication for certification policy that is helpful in world of uncertainty. When policymakers and program managers are selecting whatever CD they think is optimal, or likely to be "close" to optimal, the costs from setting a CD a bit "too soon"—before T*—are bigger than the costs from setting a CD a bit "too late"—past T*. That is, it is better, in terms of cost-effectiveness, to err a bit on the side of setting a longer CD.[49]

A major result of the baseline simulation is that T* is the same for the Government Budget and the Taxpayer Perspectives. This outcome reflects of a key feature of the model from the first-order condition (4.17) that was considered in detail (for the single recertification case). It showed that the way that recertification cost *c* and monthly benefit *m* affect T* is through the term ($\rho c/m$), i.e., only the relative values matter.[50] While the *levels* of recertification costs and effective benefits each differ between the two perspectives, the differences reflect a common factor ($1+\varepsilon$) which means the two perspectives share a common value for ($\rho c/m$). It follows then that the two perspectives must share a common T*. However,

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[49] The model in Greenfield and Persselin results in a cost curve that is asymmetric, and they recognize a similar policy implication for the timing of aircraft replacement.
[50] While the term ($\rho c/m$) captures the dependence of T* on c and m, the effect of ρ on T* appears in part through that term and in part through other terms.

because the dollar levels of effective benefits and recertification costs do differ, so too do their resulting M(T*)—but precisely by the (1+ε) factor, i.e., ($577.06/$461.65) equals 1.25.
By construction, the Social Perspective will always have the lowest effective benefit and the largest recertification cost of any of the three perspectives. Its large recertification cost reflects the inclusion of the cost paid by the client, a cost that for the baseline parameters amounts to a share of recertification cost of 15 percent ($17, inclusive of a fixed $5 out-of-pocket cost, out of $112.25). The Social Perspective's optimal CD must necessarily be the largest of the three perspectives because its (ρc/m) term is the largest. The Table shows its T* to be12 months, double the T* of the other perspectives, with an associated M(T*) of $486.28.

A discussion of the policy implications of the baseline scenario would be informed by first simulating what difference is made in the model's solution if different values for parameters were used.

Sensitivity Analysis: Effective Benefits. The first sensitivity analysis considers how an optimal CD might vary if a different figure were used for monthly benefits and all other parameters were fixed at their baseline values. While the baseline figure for *m* was $32.52 (the 17-State average), the WIC children's food package cost was estimated to be $36.27 in New York in Table 5. Using the New York figure results in a solution (not shown in a table) of 6 months for T* and $475.47 for M(T*) for the Government Budget Perspective.

It may at first seem an unusual result that the simulated T* did not vary from the baseline figure of 6 months even though *m* was increased. After all, the thrust of the CD model suggests that a higher monthly benefit would lower T* in order to detect and curtail unwarranted benefits more quickly. The explanation is that the simulation model was calibrated in units of a month. If instead the calibration was in terms of *days*, it is plausible that the number of days for T* would indeed fall in response to the use of $36.27 instead of $32.52 for *m*. A unit of a month is a more aggregate period of time, and a larger increase in *m* is needed for a change in T* to cross the threshold of a month. The exercise useflly illustrates that cross-State variation in WIC food package cost may be important for some policy issues, but not necessarily for certification policy: the size of the cross-State variation seems to be sufficiently small that it may be neglected when certification periods are set in terms of months.

Suppose instead that monthly benefits were to double, to $65.04. The results of this exercise are shown in Table 8. With an increase in effective benefits of this magnitude, T* does decrease. The Government

Budget (and Taxpayer) Perspectives put T* at 4 months, down by 2 months from the baseline case. The Social Perspective's T* is 9 months, down by 3 months from the baseline case.

A doubling of the effective benefit is not simply a sensitivity analysis that gauges the sensitivity of T* to *m*. While no State is known to have a WIC food package cost as high as $65.04 per package (i.e., per child), in every State there are those WIC households in which two WIC children (ages 1 to 4) reside. While WIC as a program is certifies and provides benefits at the level of the individual (unlike the FSP, which certifies and provides benefits at the level of household), it is still the case that a household with two WIC children can receives double the food package benefits received by a household with one child. Moreover, it is necessarily the case the two WIC children residing in the same household have the same family income month-by-month, i.e., they are eligible or ineligible jointly (in terms of income—not in terms of nutritional risk). An imaginable alternative to existing certification policy is that the WIC certification period for a WIC household could depend on the number of children (and, more generally, other participants) in the household.

Sensitivity Analysis: Overhead Rate. An arbitrary overhead rate of 100 percent was applied to total compensation (per hour) as part of estimating the agency cost of recertification. If no overhead facto were applied, the labor cost per certification would drop by half, to $38.10 [($25.40/hr)(1 ½ hr)]. T* would be expected to fall below the baseline figure of 6 months. But a more precise prediction can be made. Because a decrease in *c* by half has the same effect on the term (ρc/m) as a doubling of the monthly benefit, the T* that results when ignoring overhead is the same T* from the double-the-benefits exercise in Table 8—4 months (the M(T*) values differ between this simulation and the one in Table 8). Given continuity, some overhead factor between 0.0 and the baseline's 2.0 figure would put T* at 5 months, between the 4 month and the baseline's 6 month figures. Trial-and-error shows that if loaded labor cost is set at 1.72 of the $25.40/hour figure, T* is 6 months but if the overhead factor is set at 1.71, T* is nudged to 5 months.

Sensitivity Analysis: Excess Burden. The issue of selecting the "appropriate" or "most accurate" measure of excess burden is of no consequence for estimating T* for the first two perspectives in the table—their T* is invariant as ε changes. However, the particular value of ε that is used for simulation can matter quite a lot from the Social Perspective. After all, its baseline effective benefit figure of $8.13 figure is the simple product of effective benefits and excess burden. As the value for ε varies, effective benefits will vary in proportion—but recertification costs will vary *less* than proportionately. The component of the recertification costs paid by the client are unaffected by variation in ε. Thus, the Social Perspective is unique among the three perspectives in that changing ε changes the ratio (ρc/m) for it.

For example, Table 9 shows solutions that result from taking all other parameters at their baseline value but reducing ε from the OMB figure of 0.25 to the Jorgenson and Yun figure of 0.195. The reduction in *percentage terms* for effective benefits from $8.13 to $6.34 is relative large, while the reduction in recertification cost from $112.25 to $108.06 is relatively small, making the (ρc/m) necessarily rise. An increase in (ρc/m) drives T* to increase—in this case, to 13 months. With lower values for both effective benefits and recertification costs, the new M(T*) drops (to $455.41).

Sensitivity Analysis. Discount Rate. Table 10 simulations the results from doubling of the social discount rate, from the baseline figure of 0.2 percent per month to 0.4 percent per month, while keeping other parameters at baseline. In general, the T* from *any* perspective does depend on ρ—there is nothing in the model that makes T* "invariant" across all values of ρ. However, it turns out that a doubling of ρ does not increase T* for the Government Budget Perspective, which is still 6 months. Once again, the large, discrete nature of a one-month change (as opposed to changes measured in days) is partly responsible (along with the particular parameter values specified in the baseline). As expected, the higher discount rate does decrease the discounted minimized costs M(T*), from the baseline's $461.65 to $438.18. In contrast to the limited responsiveness of T* in the Government Budget Perspectives, T* rises for the Social Perspective: from 12 months in the baseline to no less than 16 months. Its M(T*) drops from $540.90 to $442.10. The simulation results show that an increase in ρ leads to increases in T*, sometimes so small as to be unregistered at the monthly level or by a 4-month increase above baseline.

Trial-and-error shows that T* for the Government Budget Perspective remains at 6 months for a ρ as high as 0.7 percent per month, but that T* passes the 7 month demarcation at ρ of 0.75.

Sensitivity Analysis. Income Volatility. The pair of income volatility parameters (λ, μ) in the baseline simulation were (0.07725, 0.23175) derived from the Newman data. If instead of trying to best fit the Newman data the parameters are selected to provide upper- and lower-bounds to the 3-year average figures of the state probability path $P_{EI}(t)$, different estimates for T* would result. Retaining the stipulation that the state probability path has a steady-state at 0.25 (which is λ/(λ+μ) in the model) means that μ will be 3 times the value of λ for any selected λ. Thus, this sensitivity analyis is not conducting an exercise in which one parameter is varied and all others are held constant. Income volatility is examined here through the joint variation of λ and μ.

Figure 4 shows two smooth state probability paths, a Low path and a High path, that sandwich the 3-year average state probability path of the Newman data. The Low path is generated by $\lambda = 0.06$ (and μ of 0.18), while the High path generated by $\lambda = 0.14$ (and μ of 0.42). The Low path is associated with "low" income volatility (i.e., low parameter values for the two income volatility parameters) as well as "low" values of $P_{EI}(t)$ at any given *t* along the path. Correspondingly, the High path is associated with high income volatility and high values $P_{EI}(t)$.

Table 11 shows the results for a scenario that uses the baseline parameters except for income volatility, for which low values are used. The optimal CD increases for the Government Budget Perspective is 7 months, an increase from the baseline's T* of 6 months. Similarly, T* for the Social Perspective increases, from 12 months at baseline to 14 months with low income volatility. Under high income volatility, shown in Table 12, the Government Budget Perspective's T* drop from 6 months to 4 months. The Social Perspective's T* is 7 months under high income volatility, a decrease from its baseline of 12 months.

Contrasting the M(T*) values in Table 12 with the baseline results in Table 7 shows an interesting result. Table 12 is generated by increasing income volatility, relative to the baseline. It may be unexpected that M(T*) values decrease from the baseline. (The result that optimal T* decreases for each perspective when income volatility increases is straightforward.) For the Government Budget Perspective, for example, M(T*) is $461.65 in the baseline case but drops to $414.01 in the case of high income volatility. At first this decrease in M(T*) may seem paradoxical. How can it be that increase in income volatility—which increases unwarranted benefits—could end up saving expected discount costs for the program (costs that included those unwarranted benefits)? It may seem that the paradox might be resolved by simply recognizing that T* is endogenous, which affects the point along which the new M(T) is evaluated, with T* dropping as a result of greater income volatility. However, that explanation is partial at best. After all, if the *entire* M(T) curve were to shift upwards as a result of an increase in income volatility, then even at a new, lower value of T* (at the minimum of the new M(T) curve) the value of M(T*) would *necessarily* be *higher* than the original M(T*) at the baseline income volatility. Yet the simulations show that M(T*) decreases following an increase in income volatility. What truly resolves the paradox is the recognition, from the previous section, that an increase in income volatility *twists* the M(T) curve.[51] The situation is depicted in Figure 2, which actually represents the numerical curves for

[51] Strictly speaking, the theoretical exercise in the previous section varied λ alone, holding μ constant, whereas the simulation varied λ and μ jointly. The graph corresponds to the simulation.

the Government Budget Perspective: M1(T) for the baseline and M2(T) for the "high" income volatility cases. At relatively "low" levels of T, an increase in λ lowers the M(T) curve because the unwarranted benefits effect is dominated by the recertification effect—the savings the accrue because higher income volatility raises the chance that the client is found to be ineligible at any given recertification.

Operating Characteristics in the Baseline Simulation

The Appendix presented examples of important operating characteristics for the CD model, some of which are considered here for the WIC example. The baseline scenario results in a simulated T* of 6 months for the Government Budget and Taxpayer Perspectives. Inasmuch as actual certification policy for WIC children is also 6 months, focusing on a CD of 6 months is natural. Examination of operating characteristics under a Social Perspective is not pursued.

A pair of operating characteristics of interest are the state probabilities $P_{EI}(T)$ and $P_{EE}(T)$. How many families will be income ineligible at recertification if they are recertified (at the end of) every six months? From Table 4, the state probability $P_{EI}(6)$ directly from the 3-year average figures of the Newman study is 22.0 percent.[52] The corresponding $P_{EE}(6)$ is 88.0 percent. However, the 3-year average figures reflect the vicissitudes of the stochastic income process, even though they represent some amount of smoothing through taking a 3 year average. A further smoothing is possible based on the CD model's theoretical expression for $P_{EE}(T)$ and $P_{EI}(T)$ in (4.8) and (4.9), together with the baseline's estimated values for the income volatility parameters. The resulting "fitted" values for the state probability of ineligibility is $P_{EI}(6)$ of 21.1 percent—from (0.25)[1 – exp(-(0.309)6)], which is shown as a "fitted" value in Table 4. The figure of 21.1 percent (for both 3-year average figure and the "fitted" figure) suggest that about one family out of five children will be found to be ineligible if recertifications are done every 6 months (under the maintained assumption that the Newman data can be taken as representative of income processes for WIC households with children).

For any one of those households found to be ineligible, it is unlikely that the household was ineligible in every month of the past six months. It is also unlikely that the household had just become ineligible. The $P_{EI}(t)$ path itself, over the course of the six month period, reveals the likelihood of ineligibility. An

[52] In a discrete-time context, it could be imagined that a client would apply at the beginning of month 1 based on income in month 0. If recertification were to occur monthly, i.e., if the next recertification were to occur at the end of month 1, the 3-year average data show that 10.9 of households would be ineligible at each recertification based on income in month 1 that is used at the recertification. Month 1 becomes the "new" month zero, and its figure of 10.9 percent—not the month 0 figure of 0.0 percent—is used to estimate how many families with children would be ineligible under monthly recertification. Similarly, the relevant probability of ineligibility for a 6-month recertification would be 22.0 percent, even though that figure is the *seventh* one in the table, inclusive of month 0.

estimate of Total Time Ineligible (from (A.2)) is 0.82 months—less than the equivalent of one full month of ineligibility. The corresponding figure for Total Time Eligible is 5.18 months out of the 6 month period. The Share of Time Ineligible is 13.6 percent from (A.4), i.e., 0.82 months of ineligibility is 13.6 percent of the 6 months of the certification period.

Given a steady program benefit per month, an estimated share of (non-discounted) unwarranted benefits for the *client* under a 6-month certification period is 13.6 percent of cumulative (non-discounted) benefits. That is, out of the $195.12 worth of WIC children's food package the client receives, $26.59 worth is received on an unwarranted basis.[53]

With each recertification having about a 0.20 chance of finding that the family has become ineligible, the expected number of the recertification on which the family is found to be ineligible, E[N], is 5; ignoring the integer constraint, E[N] = (1/0.2108) = 4.74 recertifications. The expected length of participation until ineligibility is detected, E[L], is 30 months (5 recertifications at 6 months each).[54] Ignoring the integer constraint, E[L] = 28.5 months using (A.7).

While about one-fifth of WIC children are found to be ineligible at recertification, it would be mistaken to infer that about one-fifth of all WIC children at a moment in time are ineligible. Those who are undergoing recertification are not a random subgroup of the caseload. On the contrary, the rest of the caseload is systematically different from those clients who are being recertified: the rest each have at shorter "times" on their recertification clocks. So long as the probability of ineligibility rises with the duration since the last recertification, the rest of the caseload has (on average) strictly *less* than a one-in-five chance of being ineligible. That is the figure of one-fifth serves as an *upper-bound* of the cross-sectional share of WIC children who are ineligible at a moment in time. The figure of one-fifth is below the figures reviewed from previous studies (which were in the range of one-quarter to one-third) that may be interpreted as providing upper-bounds on the cross-sectional share of ineligible WIC children. Recognizing the longitudinal nature of household's experiences, and that the state probability path of

[53] In practice, WIC food packages are calculated in discrete, monthly increments. However, if program benefits were dispensed bi-weekly or weekly, with CDs calibrated in interval units shorter than one month, it could be possible to save "fractions" of a month's worth of a food package by varying the CD.

[54] While the fifth recertification does not "count" in the sense of getting to participate past the fifth recertification, on which ineligibility is detected (on average), the first 6-month period of participation follows the application until the first recertification and it does "count."

ineligibility rises with time, provides enough structure on the problem to help identify a different upper-limit that happens to be less than previous estimates.[55]

That is as far as might be gone without making an additional assumptions. However, if the WIC caseload may be considered to be approximated by steady-state conditions, then there is an equivalence between two operating characteristics of interest. As shown in the Appendix, in a steady-state, the share of *time* than an individual client probability spends ineligible, over the client's CD period, turns out to be equal to the caseload's share of *clients* that are probability ineligible at a moment in time. The Share of Time Ineligible for an individual client was estimated above to be 13.6 percent. Under steady-state conditions, that is a measure of the cross-section share of WIC children who are ineligible at a moment, inclusive of those who are not undergoing recertification at that moment and have relatively low probabilities of ineligibility. It is noted that the 13.6 figure is but half of the upper-bound figures (one-quarter and one-third) provided in earlier studies.

8. Conclusion

At the outset, the paper stressed that the CD model and its results may be interpreted as a descriptive or as a prescriptive model. In addition, there are the different "perspectives" by which optimal CDs were simulated. Very different conclusions flow from the assorted interpretations and perspectives.

If the CD model is interpreted as a prescriptive model, there can be a very strong policy implications from the model's results. The baseline simulation found T* to be 6 months from the Government Budget and Taxpayer Perspectives, matching the WIC's actual certification policy for children. If policymakers and program managers are, prescriptively, "supposed" to be acting as agents of taxpayers when making program decisions, they have gotten WIC certification policy for children "about right." That result—that the model does not prescriptively suggest either a large increase or a larger decrease in WIC children's CD is, in itself, a result that is a strong policy implication.

On the other hand, some might stress the result that the Social Perspective's T* was a full 12 months at baseline. Interpreting the model prescriptively and from the viewpoint of standard benefit-cost analysis, the WIC's children certification is too short by half a year from where economic efficiency suggests it ought to be. Of course, a key assumption driving this result is that the Social Perspective treats the WIC

[55] It bears emphasizing that data for households with school-aged children, rather than children ages 1 through 4, were used to generate the Newman data.

food package as a transfer payment.[56] With little "counted" in the way of effective benefits—and then, only because of an excess burden factor—the Social Perspective's "solution" naturally strives to lengthen CD relative to the other two perspectives because recertification involves an outlay of economic resources that is large relative to the meager effective benefits at issue.

The three perspectives can not be reconciled. The paper's contribution is its recognition that the perspectives are different and its construction of simulations that respect each perspective's approach to the problem.

If the CD model is interpreted descriptively, then it may be considered a powerful and accurate model that generates (in the baseline scenario) a predicted T* that *matches* the CD the WIC program uses in actual practice. (This conclusion rests on the hypothesis that policymakers and program managers act as if they are the agent of the taxpayer, as opposed to the Social Perspective.) However, in another sense, the model's success is more limited. In the simulation that doubled the value of the WIC program benefit, T* is 4 months. If the model were an accurate description of policymakers' and program managers' behavior, then the WIC program would (already) have a 4-month certification period for those households that have two WIC children (barring other considerations from outside the simulation that suggest a CD longer than the simulation's T* may be appropriate).

One consideration for somewhat longer "optimal" CDs than the simulated T* of the model follows from the result that M(T) was asymmetric. The shape of M(T) means that lower-than-optimal CDs result in larger errors than higher-than-optimal CDs. In a world of uncertainty about the "true" values of the parameters ("true" values that might differ from those used in the simulation), a policy implication that follows from an asymmetric cost function (loss function) is that it may be cost-effective to err on the side of a longer CD.

A second, separate consideration that suggests "optimal" CDs may exceed the simulated T* is associated with interpreting the model's T* as a lower-bound on the "optimal" CD that would emerge in a fuller model. The CD model set aside issues of client access, assuming the participation by eligibles was not affected by T*. If clients have a behavioral response to variation in CD and policymakers are concerned

[56] That treatment is natural to a economist when considering how transfer payments are not counted in the National Income and Product Accounts. It is also common to ignore, or examine separately, distributional consequences when conducting benefit-cost analyses of most government projects. It is more difficult to imagine relying on "economic efficiency" approach alone to administer the WIC program, and set its certification policy, when the program itself is inherently distributional.

about client access, then a fuller model that took those effects into account would generate T* greater than those in this paper's simulations.

Taking these two considerations into account may suggest that the optimal CD for a household with two WIC children may be longer than the simulated figure of 4 months—possibly closer to (or even exceeding) the 6-month CD in actual use. But the same two considerations suggest that the optimal CD for a household with one WIC child would also be longer than its simulated figure (of 6 months). Thus, the descriptive interpretation of the model can not account for the use of a common CD for all WIC children.

Whether the model is used descriptively or prescriptively, a function of the model is to help clarify why different voices in the political process have different recommendations on CDs—and, for that matter, on other policies that affect program operations, program cost, and the lives of the low-income population the programs are meant to serve. There are many factors that affect CDs. Different recommendations follow from different impressions of the size and strength those factors and different perspectives from which to evaluate CD. The paper's results are not presented as definitive but as a first exploration on an issue of importance to policymakers, to program clients, and to the many other stakeholders with interests in the operations and effects of USDA food assistance programs.

Appendix. Operating Characteristics of the CD Model

There are various measures that are used to describe key features of how a system is operating.[57] Some are components of the CD model itself, while others are derivable from it. The operating characteristics examined in this section, which are each affected by certification policy, are:

- the optimal certification period
- expected discounted total cost
- state probabilities of eligibility and ineligibility at recertification
- total time eligible and total time ineligible
- shares of time eligible and time ineligible
- expected length of client participation
- expected caseload size (in steady-state)
- caseload shares of eligible participants and ineligible participants (in steady-state)

Operating Characteristics Contained in the CD Model. The optimal CD, given by T*, is a solution to the cost-minimizing problem, but it can also be considered an operating characteristic. Selection of T* is the one policy instrument under consideration, and selection of the CD affects every other operating characteristic in the system. A standard set of economic questions examines how T* varies in response to a change in each of the model's parameters.

Another operating characteristic Expected Discounted Total Costs—the criterion that is minimized in the CD model. There are three interesting uses for this characteristic. First, there would naturally be interest in the value that Expected Discounted Total Cost obtains at T* for a given set of model parameters. This cost-minimizing value was designated earlier by M(T*) or V(T*). Second, an investigation could examine how M(T*), rather than T*, varies in response to a change in model parameters.

Two other operating characteristics that come from the CD model itself are the state probabilities $P_{EE}(T^*)$ and $P_{EI}(T^*)$ that reflect both underlying income volatility and the length of the optimal CD.

[57] Sometimes the reliability literature describes these measures as "performance measures," but that term suggests that a change in such a measure unambiguously detects an improvement in the "performance" of the system. Economists specialize in the analysis of tradeoffs, and movement away from an optimum would improve some aspects of performance but detract from other aspects. If changes in a policy, such as certification duration, improves performance along every dimension, then the initial policy was not at yet an optimum, by definition. The more neutral term "operation characteristics" is used by this paper.

Total Time Eligible and Total Time Ineligible. The length of a participation spell for a client (or a distribution of participation spells for a set of clients) provides important information about a client's program experience. But that information is incomplete: re-entry into program participation is possible. Gottschalk and Moffitt (1994) argue that "In the presence of a population that has high entry, high exit, and high reentry rates onto and off of welfare, a better measure of welfare dependence is one that measures an individual's *total time on welfare in a fixed time interval*." (p. 38). The CD model's counterpart to the participation-based TTO concept is the expected Total Time Eligible (TTE), given by:[58]

$$(A.1)\, TTE(T) = \int_0^T P_{EE}(t)dt = \left(\frac{\mu}{\lambda+\mu}\right) T + \left(\frac{\lambda}{(\lambda+\mu)^2}\right)\left(1 - e^{-(\lambda+\mu)T}\right)$$

In the reliability literature, the expression (A.1) has been interpreted as the Expected Unit "On" Time (Barlow and Proschan, p. 80).

It is natural to specify T in (A.1) as the optimal duration T*: over the course of a single optimal certification cycle, how much of T* (in time units) is the client eligible? Unlike the operating characteristic of P_{EE}(T*), which records only a level of a state probability at the moment of recertification, TTE(T*) takes into account the entire state probability path from 0 to T*. The counterpart to TTE(T*) is Total Time Ineligible TTI(T*)

$$(A.2)\, TTI(T^*) = \int_0^{T^*} P_{EI}(t)dt = \left(\frac{\lambda}{\lambda+\mu}\right) T^* - \left(\frac{\lambda}{(\lambda+\mu)^2}\right)\left(1 - e^{-(\lambda+\mu)T^*}\right)$$

Naturally, TTE(T) + TTI(T) sum to T, for all T.

Shares of Time Eligible and Time Ineligible. The operating characteristics TTE(T) and TTI(T) are each measured in units of time. It is sometimes convenient to convert the two measures to express the time eligible and the time ineligible each as a share of the certification duration.

$$(A.3)\, STE(T) = \left(\frac{1}{T}\right)\int_0^T P_{EE}(t)dt$$

$$(A.4)\, STI(T) = \left(\frac{1}{T}\right)\int_0^T P_{EI}(t)dt$$

[58] The relationship in (A.1) is intuitive, and might be thought of as a definition rather than an actual result. However, a formal derivation is possible based on defining a variable for the (cumulative) time the client is eligible and using (A.1) to show that TTE(T) actually equals the expected value of that variable (Barlow and Proschan, pp. 80-81).

The share measures sum to 1, and hold for all T including the optimal T*.

Expected Length of Client Participation. Let the random variable L represent the length of time during which the client participates in the program. Under the assumption that the client does not depart participation until the client is detected to be ineligible at a recertification, L is the product of two other terms. One term is the certification duration, T, which is common across certification periods. The other term will be given by N, a random variable that counts the number of recertifications the client undergoes before being detected ineligible at recertification; N includes the recertification at which benefits and participation are terminated. Then L is given by

$$(A.5)\ L = NT$$

A simple question arises. If T were to double (in calendar time), would the expected length of a client's participation double as well? After all, a client is allowed to stay on the program twice as long before being recertified. Would participation length simply be proportional to T? It turns out the relationship between T and the expected length of L is positive but not proportional. T has direct participating-lengthening effect and an "indirect" participating-shortening effect through N (which depends on T).

$P_{EI}(T)$ is constant across recertifications, making the probability distribution for N geometric, though a function of T, given by:

$$(A.6)\ \Pr(N(T) = n) = (1 - P_{EI}(T))^{n-1} P_{EI}(T)$$

The probability distribution for L is correspondingly a geometric one as well, with simply different measurement units by which N is factored up to L via T (from (A.5)).

From (A.5), the expected value of L, as a function of the policy parameter T, is given by E[L(T)] = E[N(T)]T. With E[N(T)] given by $1/P_{EI}(T)$, based on (A.6) it follows that

$$(A.7)\ E[L(T)] = \frac{T}{P_{EI}(T)}$$

If the derivation of E[L(T)] were to only provide a measure of an operating charcteristics of the system, the derivation is complete.[59] However, it is of interest to use (A.7) to explore how expected participation length for a client varies with T.

[59] A similar expression for the average duration of unemployment is given by Carlson and Horrigan (1983). They showed that if P is the proportion of a cohort who enter unemployment that continues from one week to the next, then the spell length S averages 1/(1-P) weeks. A notational difference between that result and (A.7) is that 1/(1-P) is expressed using the probability of continuing rather than discontinuing as in (A.7). A substantive difference is

The central point of (A.7) is that it depends on two terms that work in opposite directions as the CD is increased. An increase in T tends to increase the expected length of participation, i.e., allowing more time between recertifications tends to increase the length of a certification period. That effect is intuitive.

The non-intuitive effect is due to $P_{EI}(T)$. An increase in T increases the probability that at the moment of recertification the client is found to be ineligible. If, at any given recertification, the chance of having participation terminated is increased, the expected length of participation tends to decrease. As time passes before recertification, the client is transitioning from a known condition of certain eligibility (at the moment of recertification) more closely to a steady-state of some probability of eligibility. As T increases, the state probability of eligibility falls and of ineligibility rises. As T increases the increase in $P_{EI}(T)$ in the denominator tends to *lower* (expected) participation length for the client.

If N were fixed as T varies, then E[L(T)] would vary simply in proportion to changes in T. However, the net effect on E[L(T)] from a change in T depends on the relative strength of the "direct" participation-lengthening effect of a longer CD and the "indirect" participation-shortening effect of greater detection of ineligibility at recertification. Differentiation of the natural log of E[L(T)], and taking an expansion of an exponential, shows

$$(A.8)\ \frac{d\ln\{E[L(T)]\}}{dT} = \frac{1}{T} - \frac{P_{EI}'(T)}{P_{EI}(T)} = \frac{1}{T} - \frac{\lambda+\mu}{e^{+(\lambda+\mu)T}-1}$$

$$= \frac{\left\{\left[1+(\lambda+\mu)T+\frac{(\lambda+\mu)^2T^2}{2!}+\frac{(\lambda+\mu)^3T^3}{3!}+\cdots\right]-1\right\}-(\lambda+\mu)T}{T\left[e^{+(\lambda+\mu)T}-1\right]} > 0$$

Thus, in the CD model, the net effect of T on expected participation length for the client is positive. Intuition suggests a positive relationship, but without taking into account the effect of T on N(T) and E[L(T)] it could mistakenly be concluded from (A.5) that the relationship between T and expected participation length is proportional. In fact, the elasticity of E[L(T)] with respect to T is positive but less than 1.

that Carlson and Harrigan utilize a "fixed" time unit (of weeks) whereas an important feature of (A.7) is that the "time unit" of T, the endogenous policy instrument, matters: it appears in the numerator (instead of a time unit of "1") and the denominator depends on T as well.

Expected Caseload Size in Steady-State. Let C(t;T) be the caseload, or a particular caseload subgroup of interest, at time t. The functional dependence on certification policy is shown by T. The equation of motion for C(t;T) is given by

$$(A.9)\ \dot{C}(t;T) = A(t) - D(t;T)$$

where A(t) are arrivals to the caseload—those who apply and are certified as eligible—and D(t) are departures from the caseload due to detection of ineligibility. Consider a steady-state in which A(t) is some constant instantaneous value of *A*. Changes in the caseload have fully adjusted to the given certification period T, making the caseload steady over time. In such a state, D(t; T) = A. This section derives the steady-state level of the caseload $C^{SS}(T)$ and relates it to T.

The conventional notion of a "cohort" of program clients refers to that subgroup of clients who entered program participation at the same time, which corresponds to A(t). To derive the steady-state caseload, it is helpful to alter the notion of a cohort to refer to all those who are recertified at the same time t. Here, the notion that each client is characterized by program "clocks" such as the client's recertification clock is borrowed from Ribar et al.[60] The number recertified at t, given by R(t;T), constitutes a "clock-cohort" each of whom share the same periodic dates of recertification. The number in the clock-cohort depends on T, the amount of time for each client's recertification period. At a given moment, the clock-cohort R(t;T) includes all clients who were new arrivals to the program T ago and who have reached their *first* recertification. That cohort contains A clients. The clock-cohort also includes clients who were new arrivals 2T ago and who are now at their *second* recertification. Each of them were eligible at their first recertification: those who were ineligible were dropped from the program. he remaining clients in the cohort number $P_{EE}(T)A$. The clock-cohort R(t;T) contains a subgroup of clients who were new arrivals 3T ago and were eligible at two past recertifications in succession. There are $P_{EE}(T)^2A$ of them. The number in the clock-cohort can be expressed, summing across all past entry cohorts, as:[61]

$$(A.10)\ R(t;T) = A + P_{EE}(T)A + P_{EE}(T)^2 A + P_{EE}(T)^3 A + \cdots = \frac{A}{1 - P_{EE}(T)} = \frac{A}{P_{EI}(T)}$$

A client may or may not be eligible at recertification. The clock-cohort R(t;T) is a count of clients undergoing recertification at *t* regardless of eligibility status at that moment. In a steady-state of homogenous clients, A(t) arrivals are constant A and each client shares the same state probability path

[60] Ribar et al. also considered the "clock" of limits on cumulative time of participation.

$P_{EI}(t)$ and faces the same policy T. Therefore, R(t;T) is (not surprisingly) itself a constant, independent of t, in steady-state.

As (A.10) shows, however, R(t;T) depends on T. An increase in T leads to an increase in $P_{EI}(T)$: if more time is allowed to pass before any given client is recertified, the chance that the client is ineligible at recertification increases. Given *A*, such an increase *lowers* the number in the clock-cohort who are being recertified at any given moment. Each entry cohort (except the first) had to pass through at least one recertification to still be a member of the clock-cohort at t. A higher probability $P_{EI}(T)$, due to a longer T, increases the chance of being terminated from the program at each of those past recertifications, resulting in a lower R(t;T).

It is a condition of steady-state that departures D(t;T) equal arrivals; departures are expressed as a function of T too. The number D(t;T) is the product of R(t;T), the number of clients who undergo recertification at t, and $P_{EI}(T)$, the probability that any one client in the clock-cohort is ineligible. The relationship D(t;T) = $P_{EI}(T)R(t;T)$, together with (x.x.), verifies that D(t;T) = A in steady-state.

Let time *t* at which R(t;T) is counted be somewhere in an interval lasting T. It is convenient to demark the start of the interval as time zero, making the interval [0,T]. Over the course of the interval, there is full "turnover" in recertification: each client undergoes recertification precisely once. Thus, the stock-flow relationship between R(t;T) and the steady-state caseload $C^{SS}(T)$ is simply the sum of R(t;T) across the interval [0,T].

$$(A.11)\ C^{SS}(T) = \int_0^T R(t;T)dt$$

Substituting (A.10) into (A.11) yields

$$(A.12)\ C^{SS}(T) = \int_0^T \frac{A}{P_{EI}(T)}dt = \frac{AT}{P_{EI}(T)}$$

An alternative derivation for $C^{SS}(T)$ is based on the cumulative flows of arrivals and departures over the length of a single certification cycle. Let CUMA(T) be cumulative arrivals, where

[61] Carlson and Horrigan used a similarly constructed geometric sequence of past cohorts of newly unemployed persons to derive a steady-state stock of the number of unemployed U as U = F/(1-P) where F is a steady inflow and P is the (weekly) probability of continuing unemployment.

$$(A.13)\ CUMA(T) = \int_0^T A(t)dt = \int_0^T Adt = AT$$

Over the same period, there is some cumulative number of departures, CUMD(T) given by

$$(A.14)\ CUMD(T) = \int_0^T D(t;T)dt = \int_0^T P_{EI}(T)R(t;T)dt = P_{EI}(T)\int_0^T R(t;T)dt = P_{EI}(T)C^{SS}(T)$$

The result in (A.14) used the result that $P_{EI}(T)$ is a constant for each member of the caseload, to factor the term out from integration, together with (A.11). In steady-state, it follows from the moment-by-moment condition that D(t) = A that cumulative arrivals and cumulative departures are equal. Equating (A.13) and (A.14) again yields (A.12).

An accounting-based approach begins with the identity:

$$(A.15)\ D(t;T) = \frac{D(t;T)}{R(t;T)} \frac{R(t;T)}{C^{SS}(T)} C^{SS}(T)$$

The first ratio is simply $P_{EI}(T)$. The second ratio is (1/T), i.e., any given clock-cohort represents (1/T) of the steady-state caseload.[62] That relationship can be derived in two steps: first, R(t;T) must be a constant because $P_{EI}(T)$—the ratio of R(t;T) to D(t;T)—is a constant and D(t;T) is a constant (equal to A); second, setting R(t;T) to its (unknown) constant value in the stock-flow relationship (A.11), and integrating, gives the relationship $C^{SS}(T)$ = R(t;T)T. The final step of using D(t;T) = *A* completes conversion of (A.15) to (A.12).

From (A.12) and (A.7), another way to express the caseload in steady-state is given by:

$$(A.16)\ C^{SS}(T) = \left(\frac{T}{P_{EI}(T)}\right) A = E[L(T)]A$$

The structure of (A.16) parallels Little's theorem, which has been described as "[o]ne of the most celebrated results in queuing theory." (Heyman and Sobel, p. 400) Little's theorem expresses the *expected "number" in a system* (e.g., bank customers in line) as the product of the expected (waiting) *time spent in the system* and the *arrival rate* of new entrants. The terms in Little's theorem are the queuing equivalents of $C^{SS}(T)$, E[L(T)] and *A*. The derivation of (A.16) is vastly simplified, compared to a proof of Little's theorem, by using the economist's notion of a steady-state and treating arrivals as a constant per unit time, *A*. Only departures D(t; T) are stochastic here. In contrast, traditional queueing theory treats not only departures but also arrivals as a stochastic process, which results in a richer model but one that is more complicated as well.

Whether the caseload is expressed by (A.12) or (A.16), there are "direct" and "indirect" effects of T on the steady-state caseload. An increase in T tends to *increase* the caseload "directly" because participants get to stay on longer when recertification occurs less frequently. This direct effect is captured by the term T. But, an increase in T tends to *decrease* the caseload: at any given recertification the probability of being detected ineligible increases. This indirect effect is captured by the terms $P_{EI}(T)$ and R(t;T). Similar effects arose when considering the expected length of participation for an individual client, an outcome that is not surprising because the steady-state caseload can be written as a function of E[L(T)], as shown in (A.16). Just as it was shown that E[L(T)] increases with T, with the direct effect dominating the indirect effect, so too does T increase the steady-state caseload—a result that follows immediately from (A.16) given that only E[L(T)] varies with T and *A* is a constant, unaffected by T.

A fitting conclusion to examining the relationship of the caseload to T is to re-consider the effects on the caseload due to short certification periods, i.e., to a "low" T (which is not quite the same thing as a decrease in T). There have been concerns in the FSP that short certification periods may diminish the caseload due to the barriers and disincentives that households face when T is low a recertification is frequent. Some clients who are eligible to apply may choose not to apply, thereby compromising client access and a purpose behind the program of providing food assistance to low-income households. This possibility could be incorporated into (A.16) supposing that A is not a constant but instead a function A(T) that increases with T. The less often participants have to recertify, some who would be eligible-nonparticipants choose instead to apply and participant, increasing A. The expression A(T) reflects the client-access issues associated with certification policy that have been set aside by the paper. Even if the effect of T on A(T) were introduced, it will still be the case that the caseload increases with an increase in T: the effect of T on A(T) would simply augment the effect of T on E[L(T)]. An avenue of future research is to consider the extent to which recertification lowers participation by *eligible* households, through A(T), and the extent to which it lowers participation by *ineligible* households. A motivation for recertification, including a short recertification, is to detect ineligible households, a goal that may conflict with client access for eligibles.

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[62] That relationship is easier to consider in discrete time (say, months) than in continuous time. If each client's certification lasts, say, 4 months then (1/4) of the caseload is recertified each month for each client in the caseload to

Caseload Shares of Eligible Participants and Ineligible Participants in Steady-State. At any given instant τ, what portion of the overall caseload is ineligible? This number is inherently different from the number that is found to be ineligible at recertification because the caseload as a whole is not recertified each instant. Let E(τ;T) and I(τ;T) be the numbers of eligible and ineligible participants, respectively, within the overall caseload at a moment in time in steady-state. The steady-state caseload shares of eligible and ineligible participants are:

$$(A.17)\, SCE^{SS}(\tau;T) = \frac{E(\tau;T)}{C^{SS}(T)}$$

$$(A.18)\, SCI^{SS}(\tau;T) = \frac{I(\tau;T)}{C^{SS}(T)}$$

While these shares would be steady across time (in steady-state), and therefore independent of what particular instant they are measured, the notation for τ is preserved to emphasize that the caseload is being examined in cross-section rather than longitudinally.

Let *t* represent *duration* of time for a client since certification or the last recertification, with *t* somewhere in the interval [0, T]. Thus, a given instant τ represents different *t* to different clients. Let the number of clients in the cohort of duration *t* be given by Y(t). In steady-state, the clients' durations are spread uniformly across [0, T], making the number Y(t) equal to (C^{SS}/T). That is, in steady-state, there are Y(t) clients for whom t = T: for them, recertifications are immediately due and there are (C^{SS}/T) of these clients. As well, there are also Y(t) clients for whom t = (T/2): they are precisely half-way from their last recertification to their next recertification, and there are (C^{SS}/T) of them. And so forth for every possible duration between 0 and T.

At any given instant, all the participants are somewhere along their state probabilities paths of eligibility and ineligibility P_{EE}(t) and P_{EI}(t) which, for each client, depend on the duration since recertification. The number of participants of a given cohort, Y(t), who are ineligible at an instant is given by P_{EI}(t)Y(t). Aggregating across all cohorts gives

$$(A.19)\; I(\tau;T) = \int_0^T P_{EI}(t)Y(t)dt = \int_0^T P_{EI}(t)\left(\frac{C^{SS}(T)}{T}\right)dt = \left(\frac{C^{SS}(T)}{T}\right)\int_0^T P_{EI}(t)dt$$

From (A.18) and the definition (A.17) it follows that the share of the caseload that is ineligible in steady-state is given by

be recertified in a four-month period.

$$(A.20)\, SCI^{SS}(\tau;T) = \frac{1}{T}\int_{0}^{T} P_{EI}(t)dt$$

The expression for STI(T) in (A.4) and the expression for SCI(τ;T) in (A.20) are equal. This result shows that, in steady-state, there is an equivalency between ineligibility at the client level and at the caseload level. The share of *time* than an *individual* client probabilistically spends ineligible, over the client's certification duration period turns out to be equal to the caseload's share of *clients* that are probabilistically ineligible (eligible) at a moment in time. This equivalency was derived here as a result of conditions and characterizations of steady-state, though some may consider that equivalence to be a short-hand definition of what one means by the term "steady-state."

References

Ballard, Charles. "The Marginal Efficiency Cost of Redistribution." American Economic Review Vol. 78 No. 5 (December, 1983) 1019-1033.

Barlow, Richard and Frank Proschan. *Mathematical Theory of Reliability*. Philadelphia: Society for Industrial and Applied Mathematics, 1996.

Bitler, Marianne, Janet Currie, and John Karl Scholz. "WIC Eligibility and Participation." *Journal of Human Resources* 38 (Supplement 2003), 1139-1179.

Blank, Rebecca and Patricia Ruggles. "When Do Women Use Aid to Families with Dependent Children and Food Stamps? The Dynamics of Eligibility versus Participation." Journal of Human Resources 35 (Winter 1996), 57-89.

Boardman, Anthony, David Greenberg, Aidan Vining, and David Weimer. *Cost-Benefit Analysis: Concepts and Practice*. Upper Saddle River, NJ: Prentice Hall, 1996.

Boskin, Michael and Frederick Nold. "A Markov Model of Turnover in Aid to Families with Dependent Children." *Journal of Human Resources* Vol. 10, No. 4 (Autumn, 1975) 467-481.

Burstein, Nancy. *Dynamics of the Food Stamp Program as Reported in the Survey of Income and Program Participation*. Report to the U.S. Department of Agriculture. Cambridge, MA: Abt Associates, Inc., January 1993.

Capital Planning and IT Investment Committee, Federal Chief Information Officers Council. *ROI and the Value Puzzle*. April 1999.

Carlson, John and Michael Horrigan. "Measures of Unemployment Duration as Guides to Research and Policy: Comment." *American Economic Review* Vol. 73 No. 5 (December 1983) 1143-1150.

Carlton, Dennis and Jeffrey Perloff. *Modern Industrial Organization*. 2nd edition. New York: HarperCollins College Publishers, 1994.

Cody, Scott, Phil Gleason, Bruce Schechter, Miki Satake, and Julie Sykes. *Food Stamp Program Entry and Exit: An Analysis of Participation Tends in the 1990s*. CCR-8. Washington, DC: U.S. Department of Agriculture, July 2005.

Davis, David and Ephraim Leibtag. Interstate Variation in WIC Food Package Costs: The Role of Food Prices, Caseload Composition, and Cost-Containment Practices . FANRR 41. Washington DC: U.S. Department of Agriculture, January 2005.

Farrell, Mary, Michael Fishman, Matthew Langley, and David Stapleton. *The Relationship of Earnings and Income to Food Stamp Participation.* E-FAN-03-011. Washington, DC: U.S. Department of Agriculture, April 2006.

Foster, E. Michael and E. Wayne Holden. "Benefit-cost analysis in the evaluation of child welfare programs." *Focus* Vol. 23, No. 1 (Winter, 2004)

Gabor, Vivian, Brooke Layne Hardison, Christopher Botsko, and Susan Bartlett. Food Stamp Program Access Study: Local Office Policies and Practices. E-FAN-03-013. Washington, DC: U.S. Department of Agriculture, December 2003.

Gleason, Philip, Peter Schochet and Robert Moffitt. "The Dynamics of Food Stamp Program Participation in the Early 1990s." Report to the U.S. Department of Agriculture. Princeton, NJ: Mathematica Policy Research, Inc., April 1998.

Gordon, Anne, Kimball Lewis, and Larry Radbill. *Income Variability Among Families with Pregnant Women, Infants, or Young Children*. Report to the U.S. Department of Agriculture. Princeton, NJ: Mathematica Policy Research, Inc., January 1997.

Gottschalk, Peter and Robert Moffitt. "Welfare Dependence: Concepts, Measures, and Trends" *American Economic Review* Vol. 84 Nol. 2 (May 1994) 38-42.

Greenfield, Victoria and David Persselin. *An Economic Framework for Evaluating Military Aircraft Replacement*. Santa Monica, CA: RAND, 2002.

Hamilton, William, Nancy Burstein, Margaret Hart, Susan Bartlett, Mary Beth Sullivan, and Ruy Teixeira. *Factors Affecting Food Stamp Certification Cost, Volume 1*. Report to U.S. Department of Agriculture. Cambridge, MA: Abt Associates, Inc., November 1989.

Heyman, Daniel and Matthew Sobel. *Stochastic Models in Operations Research: Volume I, Stochastic Processes and Operating Characteristics*. Mineola, New York: Dover Publications, Inc. (1982)

Howard, Ronald. *Dynamic Programming and Markov Processes*. Camridge, MA: M.I.T. Press, 1960.

Jorgenson, D.W., J.J. McCall, and R. Radner. *Optimal Replacement Policy*. Amsterdam: North-Holland Publishing Company, 1967.

Lancaster, Tony. *The Econometric Analysis of Transition Data*. Cambridge, U.K.: Press Syndicate of the University of Cambridge, 1990.

Logan, Christopher, William Rhodes, and Joseph Sabia. Food Stamp Program Costs and Error Rates, 1989-2001. CCR-15. Washington, DC: U.S. Department of Agriculture, January 2006.

Kabbani, Nader and Parke Wilde. "Short Recertification Periods in the U.S. Food Stamp Program." *Journal of Human Resources* 38 (Supplement, 2003), 1112-1138

Klerman, Jacob and Arleen Leibowitz. "Child Care and Women's Return to Work After Childbirth." *American Economic Review*. Vol. 80 No. 2 (May 1990) 284-288.

Kuchak, JoAnne and Nancy Pindus. *The WIC Dynamics Study, Volume I, Final Report*. Report to U.S. Department of Agriculture. Calverton, MD: Macro International, Inc., February 1995.

MaCurdy, Thomas and Grecia Marrufo. *The MID-SIPP Model: A Simulation Approach for Projecting Impacts of Changes in the Food Stamp Program*. CCR-24. Washington, DC: U.S. Department of Agriculture, October 2006.

McCall, Mary and Barbara Keir. *Survey of the Public Health Nutrition Workforce, 1999-2000*. Report to the U.S. Department of Agriculture. Alexandria, VA: January 2003.

McKernan, Signe-Mary and Caroline Ratcliffe. *Transition Events in the Dynamics of Poverty*. Washington, D.C.: The Urban Institute, September 2002.

Mallar, Charles, Stuart Kerachsky, Craig Thornton, and David Long. *Evaluation of the Economic Impact of the Job Corps Program, Third Follow-Up Report, Executive Summary*. Report to the U.S. Department of Labor. Princeton, NJ: Mathematica Policy Research, Inc., September 1982.

Newman, Constance. The Income Volatility See-Saw: Implications for School Lunch. ERR-23. Washington, DC: U.S. Department of Agriculture, August 2006.

Ponza, Michael James Ohls, Lorenzo Moreno, Amy Zambrowski, and Rhoda Cohen. *Customer Service in the Food Stamp Program*. Report to U.S. Department of Agriculture. Princeton, NJ: Mathematica Policy Research, Inc., July 1999.

Radner, J., and D.W. Jorgenson. "Optimal Replacement and Inspection of Stochastically Failing Equipment," in Arrow, Kenneth, Samuel Karlin and Herbart Scarf, eds. *Studies in Applied Probability and Management Science*. Stanford, CA: Stanford University Press, 1962.

Ribar, David, Marilyn Edelhoch and Qiduan Liu. *South Carolina Food Stamp and Well-Being Study: Transitions in Food Stamp and TANF Participation and Employment Among Families With Children*. CCR-17. Washington, DC: U.S. Department of Agriculture, April 2006a.

Ribar, David, Marilyn Edelhoch and Qiduan Liu. *South Carolina Food Stamp and Well-Being Study: Transitions in Food Stamp Participation and Employment Among Adult-Only Households*. CCR-18. Washington, DC: U.S. Department of Agriculture, April 2006b.

Samuelson, Paul. "Economics of Forestry in an Evolving Society." Reprinted in Gopalakrishnan, Chennet, ed. *Classic Papers in Natural Resource Economics*. New York: St. Martin's Press, 2000.

U.S. Congressional Budget Office. *Improving Work-Welfare Programs: Issues and Options*. Washington, DC: Congressional Budget Office, 1983.

U.S. Congressional Budget Office. *Work-Related Programs for Welfare Recipients*. Washington, DC: Congressional Budget Office, April 1987.

U.S. Department of Agriculture, Food and Nutrition Service, Office of Analysis, Nutrition and Evaluation. *WIC Participant and Program Characteristics 2000*, WIC-02-PC, by Susan Bartlett, Ramona Olvera, Nicole Gill, and Michele Laramie. Project Officer, Julie Kresge. Alexandria, VA: March 2002.

U.S. Department of Agriculture, Food and Nutrition Service, Office of Analysis, Nutrition, and Evaluation. *National Survey of WIC Participants, 2001 Final Report*, by Nancy Cole, David Hoaglin, and John Kirlin. Project Officer, Julie Kresge. Alexandria, VA: 2001.

U.S. Department of Agriculture, Food and Nutrition Service. *Characteristics of Food Stamp Households: Fiscal Year 2004*. Report No. FSP-05-CHAR. September, 2005.

U.S. Department of Agriculture, Food and Nutrition Service, Office of Analysis, Nutrition and Evaluation. *WIC Staffing Data Collection Project*. WIC-05-WS. Project Officer: Ed Herzog. Alexandria, VA: January 2006.

U.S. Department of Agriculture, Food and Nutrition Service, Office of Analysis, Nutrition and Evaluation. *WIC Participant and Program Characteristics 2004*, WIC-04-PC, by Susan Bartlett, Ellen Bobronnikov, and Nicole Pacheco, et al. Project Officer, Fred Lesnett. Alexandria, VA: March 2006.

U.S. Government Accountability Office. *Tax Policy: Summary of Estimates of the Costs of the Federal Tax System.* GAO-05-878. August 2005.

U.S. Office of Management and Budget. *Circular No. A-94 (Revised): Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs.* October 1992.

U.S. Office of Management and Budget. *Circular No. A-76 (Revised) Performance of Commercial Activities.* May 2003.

U.S. Office of Management and Budget. *Circular No. A-94, Appendix C: Discount Rates for Cost-Effectiveness, Lease Purchase, and Related Activities.(Revised)* January 2006.

Ver Ploeg, Michele and David Betson, Eds. *Estimating Eligibility and Participation for the WIC Program: Final Report*. Washington, D.C. The National Academies Press, 2003.

Internet References

Clinton County Health Department (New York)
http://www.clintoncountygov.com/Departments/Health/wic.html

Fairfax County Health Department (Virginia)
http://www.fairfaxcounty.gov/hd/wic/

Minnesota Department of Health
http://www.health.state.mn.us/divs/fh/wic/aboutwic/wicappt.html

Michigan Department of Community Health
http://www.michigan.gov/mdch

Rice County, Minnesota
http://www.co.rice.mn.us/personnel/jobs.php

St. Charles County Division of Public Health (Missouri)
http://www.scchealth.org/docs/ph/ph_docs/phphs/wiccert.html

U.S. Bureau of Labor Statistics, National Compensation Survey
http://www.bls.gov/ncs/home.htm#data

U.S. Department of Agriculture, Food and Nutrition Service
www.fns.usda.gov/pd/wic_monthly.htm

U.S. Department of Agriculture, Food and Nutrition Service
http://www.fns.usda.gov/pd/wisummary.htm

Utah County Department of Health (Utah)
http://www.co.utah.ut.us/Dept/HealthWIC/Index.asp

Table 1. Distribution of Certification Periods of Participating FSP Households, by Selected Characteristics, Fiscal 2004

Months in Certification Period	Total Households (percent)	Households with Countable Earned Income (percent)	Households with Elderly (percent)
1	0.2	0.2	0.1
2	0.4	0.2	0.1
3	2.7	2.3	0.5
4	1.6	1.4	0.2
5	1.6	1.9	0.4
6	35.4	54.1	8.6
7	2.1	2.8	0.6
8	0.6	0.7	0.4
9	0.3	0.3	0.2
10	0.6	0.6	0.5
11	1.3	1.0	1.8
12	43.4	33.2	57.7
13+	9.4	1.3	28.7
Unknown	0.2	0.1	0.3
Average certification period	10 months	8 months	16 months

Source: *Characteristics of Food Stamp Households: Fiscal Year 2004* (U.S. Department of Agriculture, 2005), Table A-12.

Table 2. Benefit-Cost Analysis Comparing "Never Recertify" and "Recertify Once Now" Policies in Steady-State

	Client Perspective	**Taxpayer Perspective**	**Social Perspective**
Never Recertify Policy			
(1) Recertification Costs	0	0	0
(2) Program Benefits	$+\left(\frac{1+r}{r}\right)M$	$-(1+\varepsilon)\left(\frac{1+r}{r}\right)M$	$-\varepsilon\left(\frac{1+r}{r}\right)M$
(3) E[Net Present Value] (3) = (1)+(2)	$+\left(\frac{1+r}{r}\right)M$	$-(1+\varepsilon)\left(\frac{1+r}{r}\right)M$	$-\varepsilon\left(\frac{1+r}{r}\right)M$
Recertify Once Policy			
(4) Recertification Costs	$-C_C$	$-(1+\varepsilon)C_A$	$-\left[(1+\varepsilon)C_A + C_C\right]$
(5) Program Benefits, if Ineligible (probability P)	0	0	0
(6) Program Benefits, if Eligible (probability (1-P))	$+\left(\frac{1+r}{r}\right)M$	$-(1+\varepsilon)\left(\frac{1+r}{r}\right)M$	$-\varepsilon\left(\frac{1+r}{r}\right)M$
E[Unwarranted Benefits]	$+\left(\frac{1}{r}\right)PM$	$-(1+\varepsilon)\left(\frac{1}{r}\right)PM$	$-\varepsilon\left(\frac{1}{r}\right)PM$
E[Warranted Benefits]	$+\left[M+\left(\frac{1}{r}\right)(1-P)M\right]$	$-(1+\varepsilon)\left[M+\left(\frac{1}{r}\right)(1-P)M\right]$	$-\varepsilon\left[M+\left(\frac{1}{r}\right)(1-P)M\right]$
(7) E[Net Present Value] (7) = (4) + [P(5)+(1-P)(6)]	$+\left(\frac{1+r}{r}\right)(1-P)M - C_C$	$-(1+\varepsilon)\left[\left(\frac{1+r}{r}\right)(1-P)M + C_A\right]$	$-\varepsilon\left(\frac{1+r}{r}\right)(1-P)M - \left[(1+\varepsilon)C_A + C_C\right]$
Benefit-Cost Analysis of Recertify Once Policy vs. Never Recertify Policy			
(8) E[Net Present Value] (8) = (7) – (3)	$-\left(\frac{1+r}{r}\right)PM - C_C$	$+(1+\varepsilon)\left[\left(\frac{1+r}{r}\right)PM - C_A\right]$	$+\varepsilon\left(\frac{1+r}{r}\right)PM - \left[(1+\varepsilon)C_A + C_C\right]$

Table 3. Income Eligibility Patterns of Families with Children Ages 1 to 4, 1990-92

	Eligible in No Months	Eligible in Some Months	Eligible in All Months	Row totals
Annually Ineligible	40.9	17.0	0.0	57.9
Annually Eligible	0.0	15.7	26.5	42.2
Column totals	40.9	59.2		100.1

Source: U.S. Department of Agriculture (1997), Table III.5

Table 4. State Probability Paths of Income Ineligibility, 1996-98

(185 percent of poverty, households with school-aged children)

				Average, 1996-98	Fitted
Month	1996	1996	1998		
0	0.0	0.0	0.0	0.0	0.0
1	13.7	11.8	7.2	10.9	6.6
2	12.9	13.1	11.1	12.4	11.5
3	17.6	21.1	17.0	18.6	15.1
4	19.5	18.4	16.8	18.2	17.7
5	20.3	19.8	19.0	19.7	19.7
6	22.1	23.8	20.0	22.0	21.1
7	23.0	21.6	18.8	21.1	22.1
8	22.2	22.0	20.6	21.6	22.9
9	23.1	23.5	23.8	23.5	23.5
10	25.9	25.3	22.5	24.6	23.9
11	23.3	23.2	23.6	23.4	24.2

Source: Newman (2006), Tables 10-12 and ERS analysis

Table 5. Estimated WIC Children's Food Package Cost, by State, 1997-1999

State	**Cost of WIC Children's Food Package**
CA	$34.71
CO	$33.52
FL	$33.15
GA	$31.12
IA	$30.36
IL	$32.79
KS	$32.64
MA	$34.27
MI	$31.60
MN	$30.74
MO	$31.25
NY	$36.27
PA	$32.18
TN	$32.78
TX	$31.24
WA	$34.24
WI	$30.04
Average	$32.52

Source: Davis and Leibtag (2005) and ERS analysis

Table 6. Set of Parameters Values for Baseline Simulation of CD Model.

Parameter	Baseline Values
Exit rate per month (λ)	7.725 percent
Re-entry rate per month (μ)	23.175 percent
Benefit per month (m)	$32.52
Staff time to re-certify	1 ½ hour
Loaded hourly labor cost	$50.80/hour
Client time to recertify	2 hour
Value of time to client	$6.00/hour
Out-of-pocket cost for client	$5.00
Marginal Excess Burden (ε)	25 percent
Social discount rate per month	0.2 percent

Table 7. Results of CD Model Simulation: Baseline

	Government Budget Perspective	Taxpayer Perspective	Social Perspective
Recertification Cost	$76.20	$95.25	$112.25
Effective Benefit	$32.52	$40.65	$8.13
Optimal CD (T*) (end of month)	6 months	6 months	12 months
Optimized Total Cost M(T*)	$461.65	$577.06	$486.28

Table 8. Results of CD Model Simulation: Baseline, Except Double Effective Benefits

	Government Budget Perspective	Taxpayer Perspective	Social Perspective
Recertification Cost	$76.20	$95.25	$112.25
Effective Benefit	$65.04	$81.30	$16.26
Optimal CD (T*) (end of month)	4 months	4 months	9 months
Optimized Total Cost M(T*)	$560.90	$663.30	$540.90

Table 9. Results of CD Model Simulation: Baseline, Except Excess Burden Decreased to 19.5 Percent

	Government Budget Perspective	Taxpayer Perspective	Social Perspective
Recertification Cost	$76.20	$91.06	$108.06
Effective Benefit	$32.52	$38.86	$6.34
Optimal CD (T*) (end of month)	6 months	6 months	13 months
Optimized Total Cost M(T*)	$461.65	$551.67	$455.41

Table 10. Results of CD Model Simulation: Baseline, Except Discount Rate Raised to 0.004

	Government Budget Perspective	Taxpayer Perspective	Social Perspective
Recertification Cost	$76.20	$95.25	$112.25
Effective Benefit	$32.52	$40.65	$8.13
Optimal CD (T*) (end of month)	6 months	6 months	16 months
Optimized Total Cost M(T*)	$438.18	$547.72	$442.10

Table 11. Results of CD Model Simulation: Baseline Case, Except Low Income Volatility

	Government Budget Perspective	Taxpayer Perspective	Social Perspective
Recertification Cost	$76.20	$95.25	$112.25
Effective Benefit	$32.52	$40.65	$8.13
Optimal CD (T*) (end of month)	7 months	7 months	14 months
Optimized Total Cost M(T*)	$485.98	$607.47	$492.23

Table 12. Results of CD Model Simulation: Baseline Case, Except High Income Volatility

	Government Budget Perspective	Taxpayer Perspective	Social Perspective
Recertification Cost	$76.20	$95.25	$112.25
Effective Benefit	$32.52	$40.65	$8.13
Optimal CD (T*) (end of month)	4 months	4 months	7 months
Optimized Total Cost M(T*)	$414.01	$517.51	$474.58

Figure 1. M(T) is Sum of E[PV(Recertification Cost(T)] and E[PV(Unwarranted Benefits(T)]

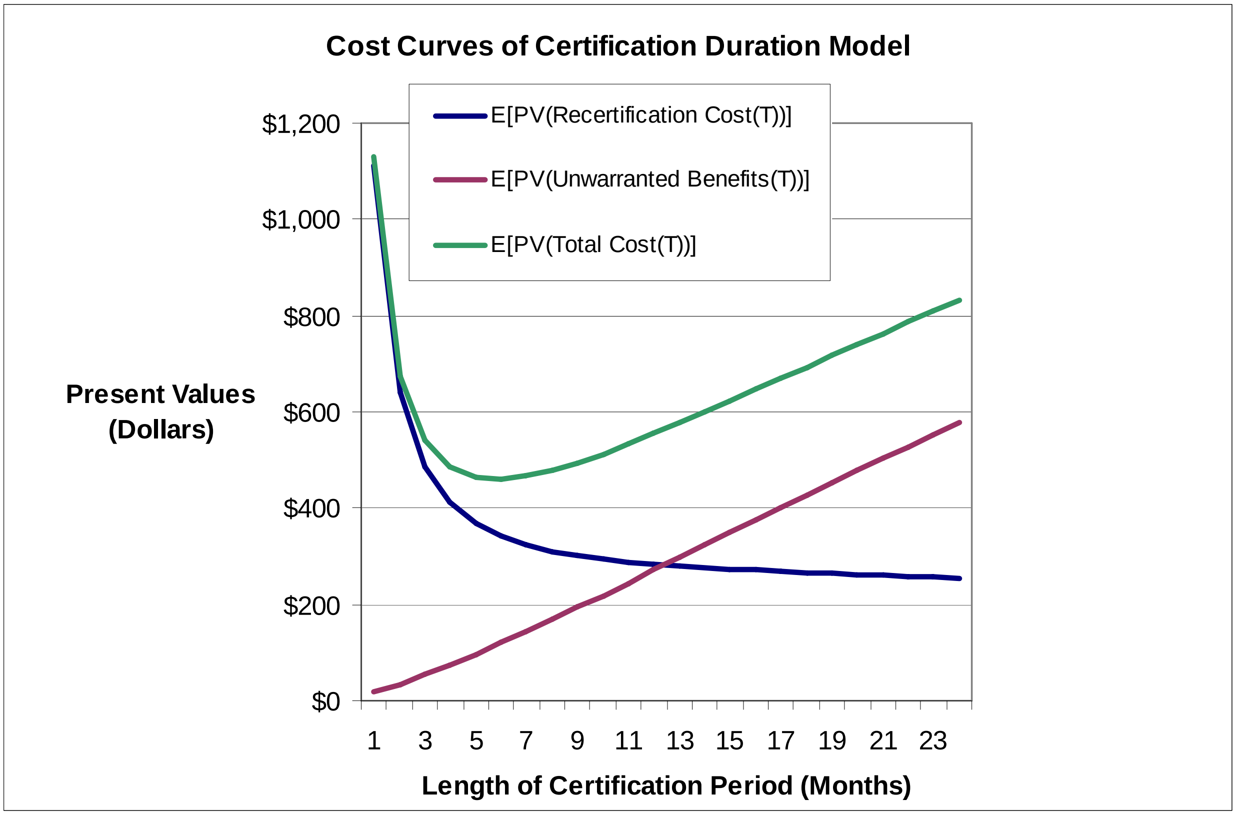

Figure 2.

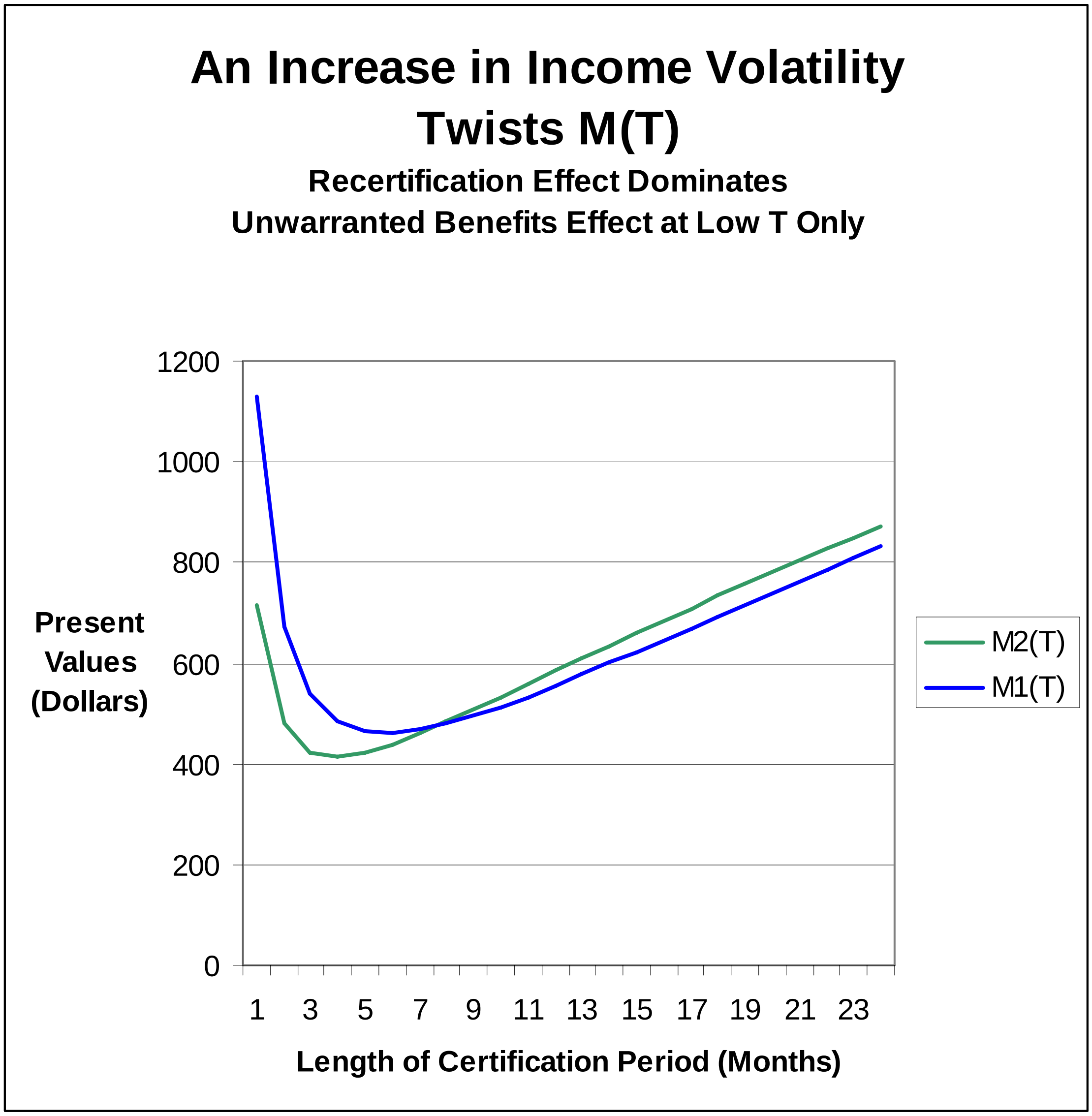
An Increase in Income Volatility
Twists M(T)
Recertification Effect Dominates
Unwarranted Benefits Effect at Low T Only
1200
1000
800
600
400
200
0
Present
Values
(Dollars)
1
3
5
7
9
11
13
15
17
19
21
23
Length of Certification Period (Months)
M2(T)
M1(T)

Figure 3.

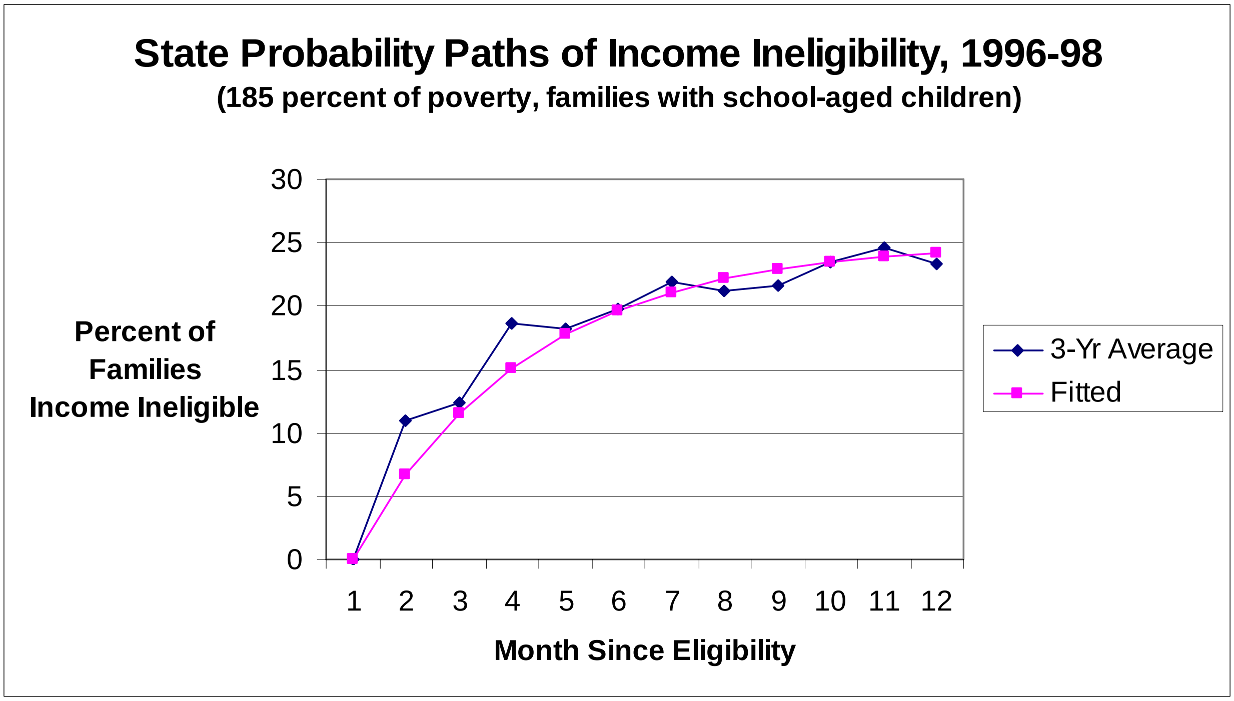

Figure 4.